Hybrid-Parallel Parameter Estimation for Frequentist and Bayesian Models

Parameswaran Raman

Ph.D. Dissertation Defense

University of California, Santa Cruz

December 2, 2019

Motivation

Data and Model grow hand in hand

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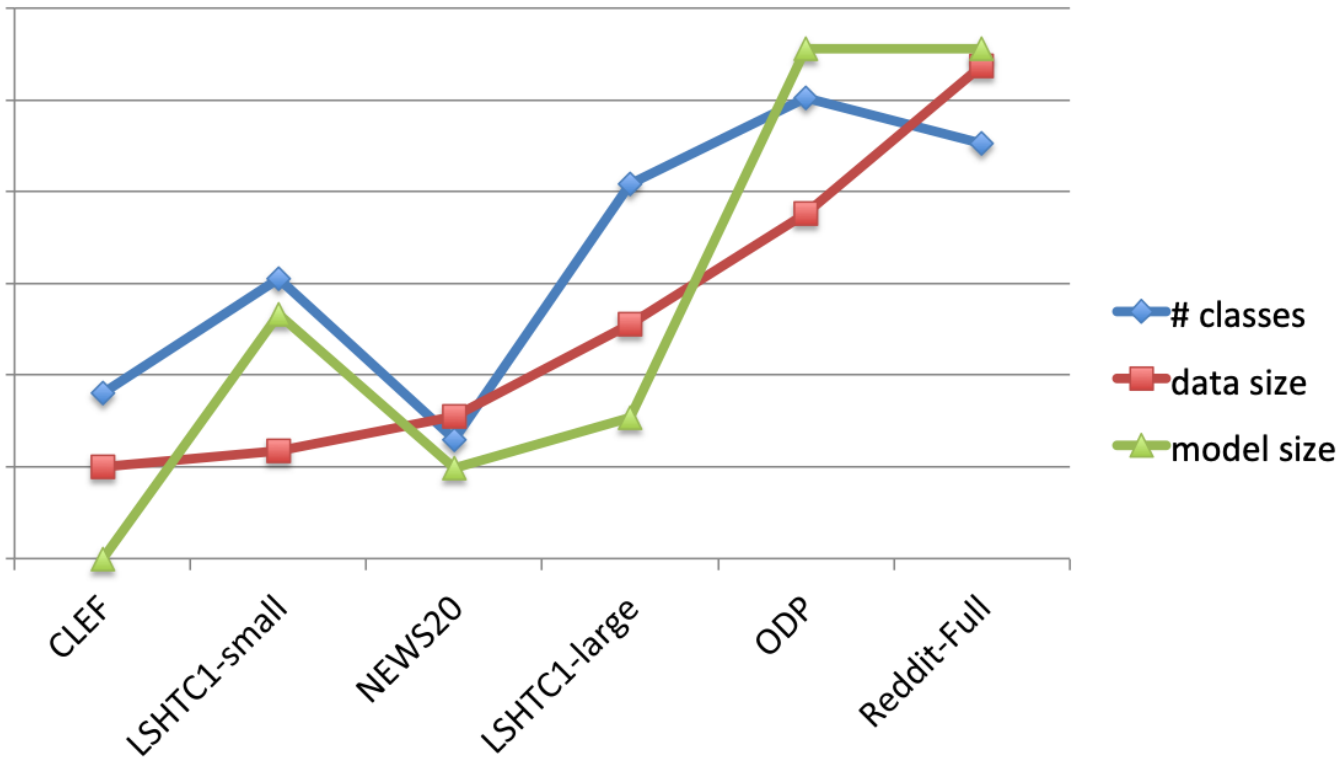
e.g. Extreme Multi-class classification



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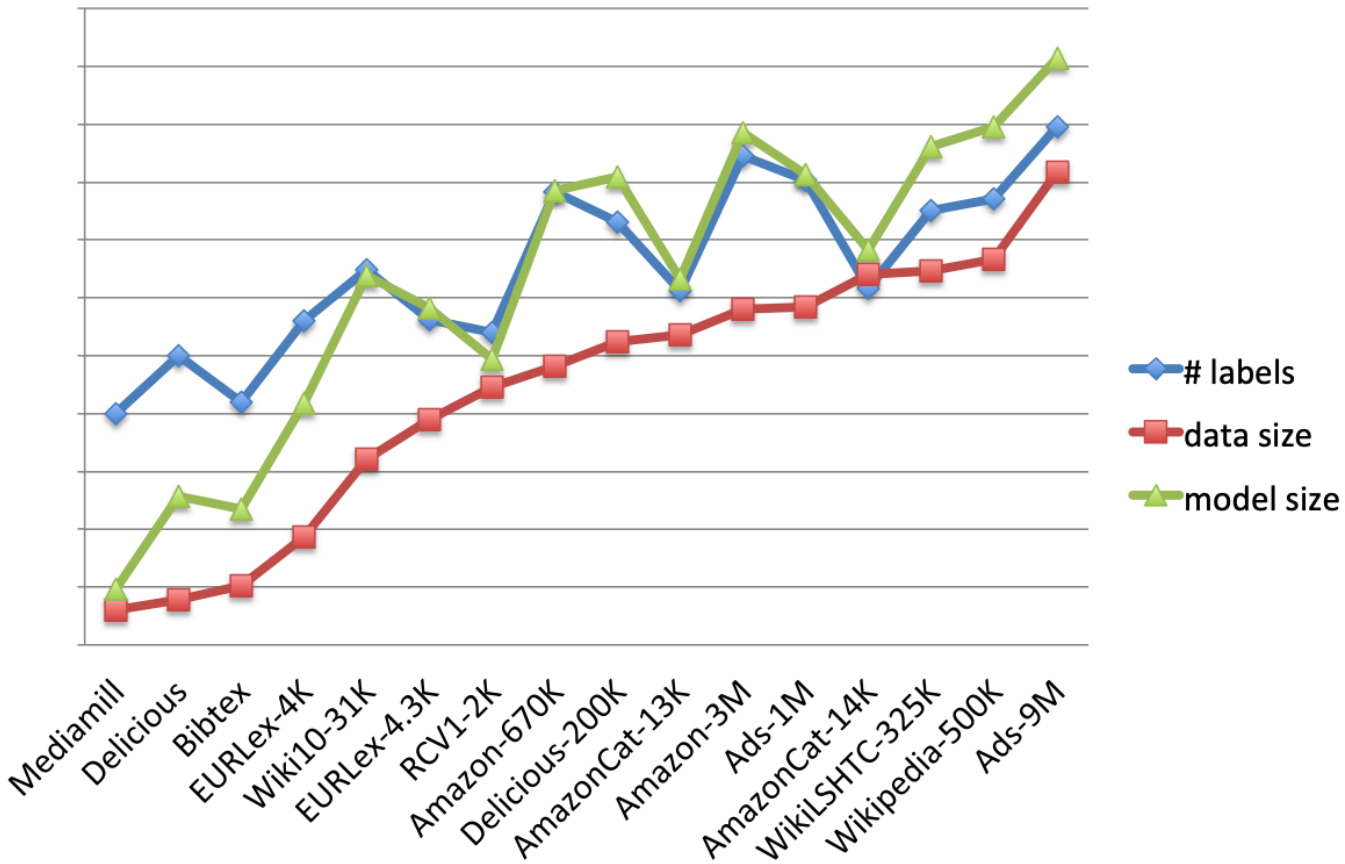

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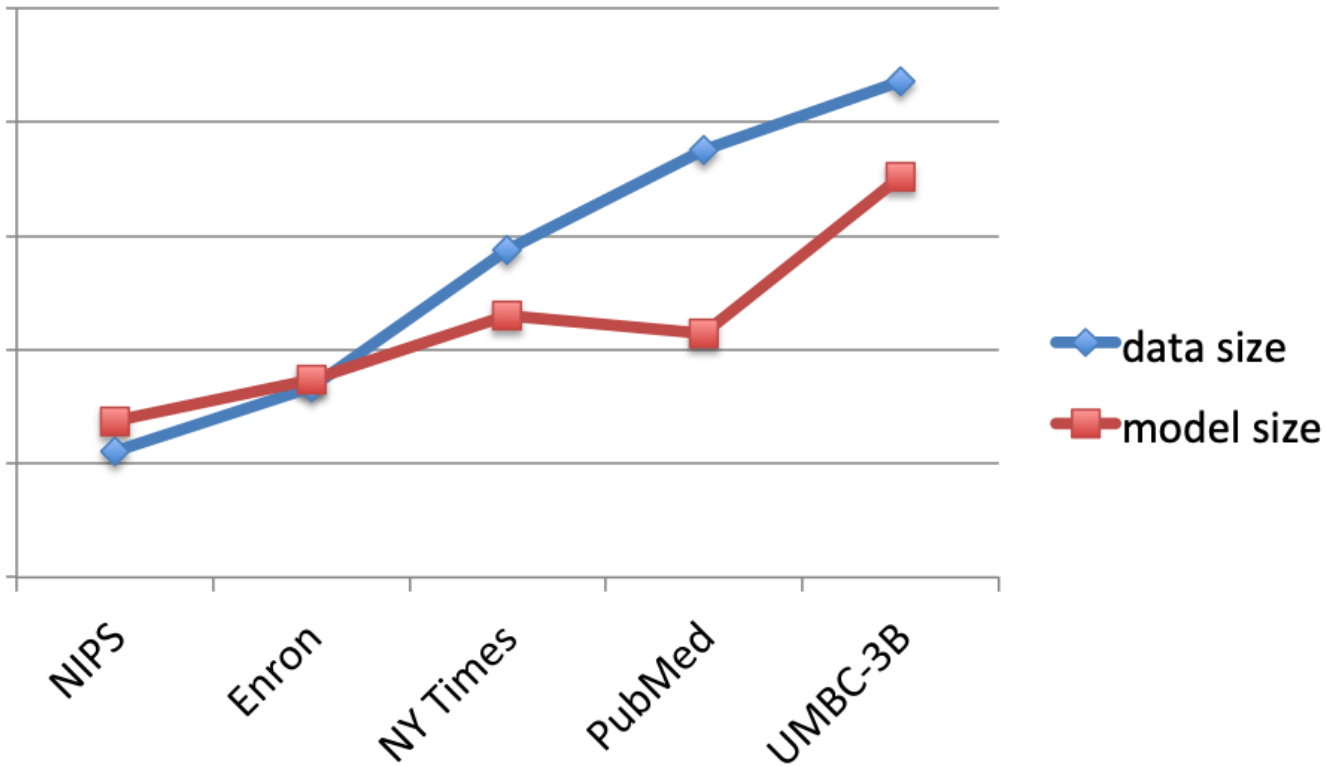
Data and Model grow hand in hand

e.g. Topic Models



Challenges in Parameter Estimation

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Hybrid-Parallel algorithms for parameter estimation!

Outline

1. Introduction
2. Distributed Parameter Estimation
3. Thesis Roadmap
   - Multinomial Logistic Regression (KDD 2019)
   - Mixture Models (AISTATS 2019)
   - Latent Collaborative Retrieval (NIPS 2014)
4. Conclusion and Future Directions
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- θ are the model parameters
- $loss(\cdot)$ to quantify model's performance
- regularizer $\mathcal{R}(\theta)$ to avoid over-fitting (penalizes complex models)

Frequentist Models

Frequentist Models

- Binary Classification/Regression
- Multinomial Logistic Regression
- Matrix Factorization
- Latent Collaborative Retrieval
- Polynomial Regression, Factorization Machines

Bayesian Models

Bayesian Models

$$\underbrace{p(\theta|\mathbf{X})}_{\textit{posterior}} = \frac{\overbrace{p(\mathbf{X}|\theta)}^{\textit{likelihood}} \cdot \overbrace{p(\theta)}^{\textit{prior}}}{\underbrace{\int p(\mathbf{X}, \theta) d\theta}_{\text{marginal likelihood (model evidence)}}}$$

- prior plays the role of regularizer $\mathcal{R}(\theta)$
- likelihood plays the role of empirical risk

Bayesian Models

- Gaussian Mixture Models (GMM)
- Latent Dirichlet Allocation (LDA)

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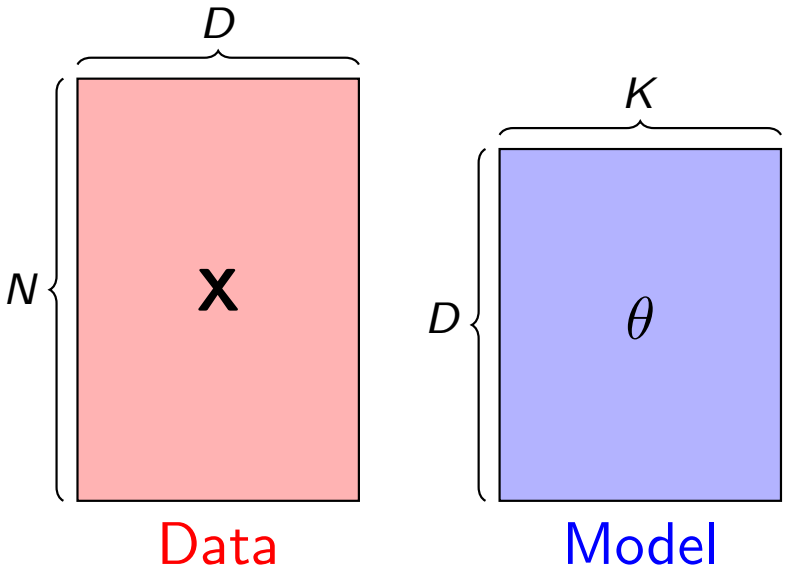
Focus on Matrix Parameterized Models



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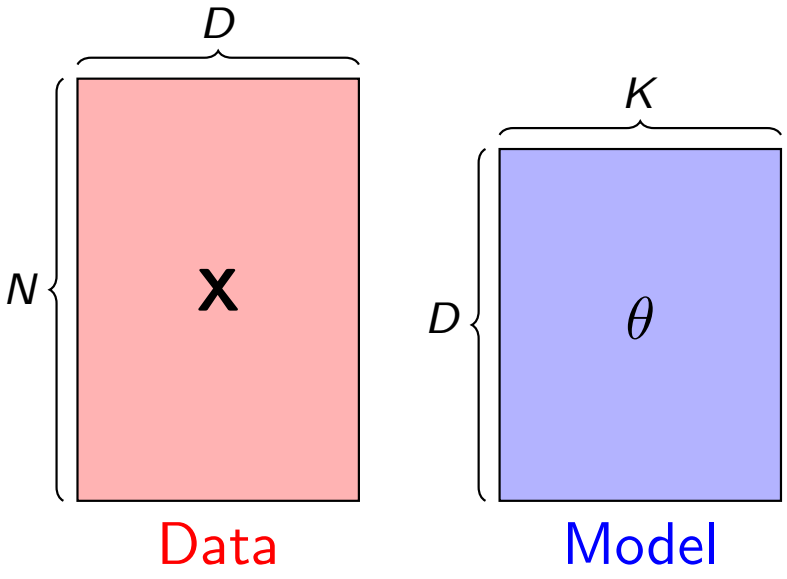

What if these matrices do not fit in memory?

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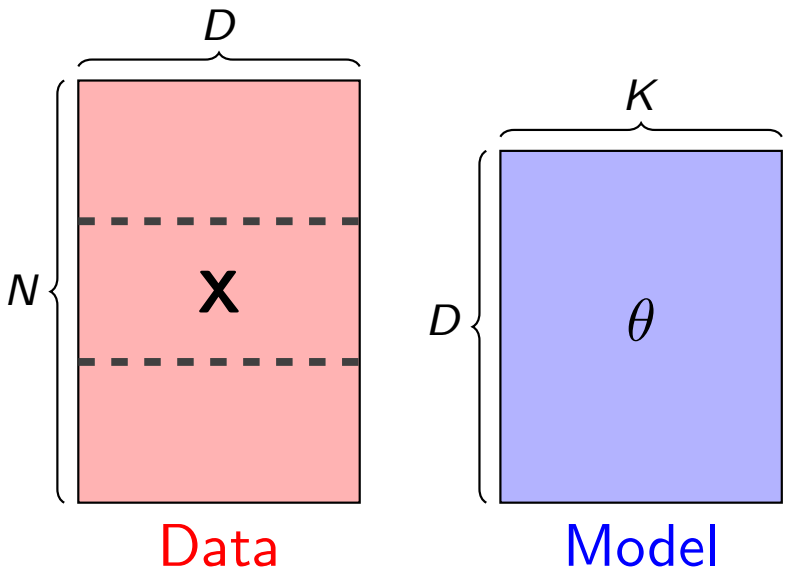
Distributed Parameter Estimation

Distributed Parameter Estimation

Data parallel



e.g. L-BFGS

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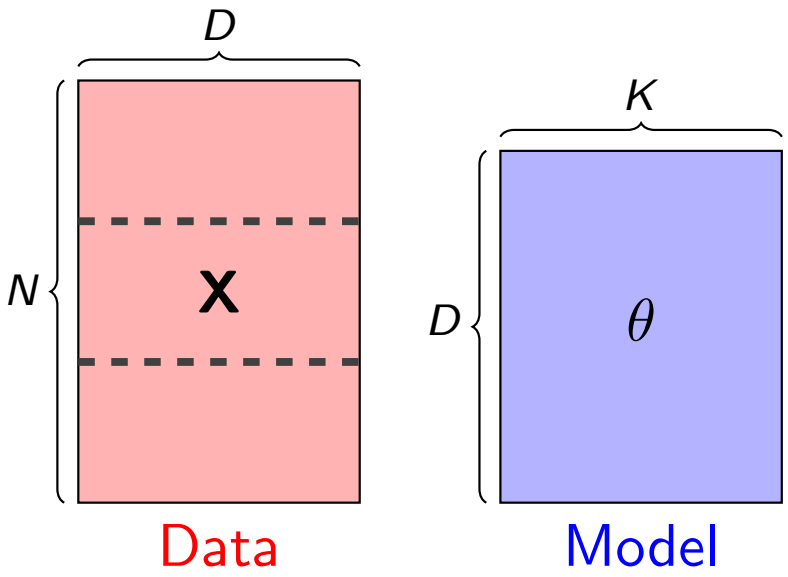

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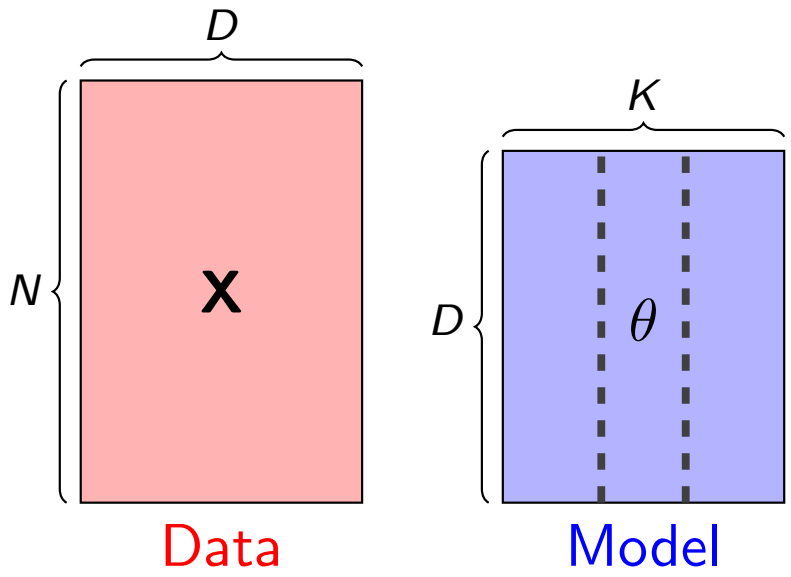
Model parallel



e.g. LC [Gopal et al., 2013]

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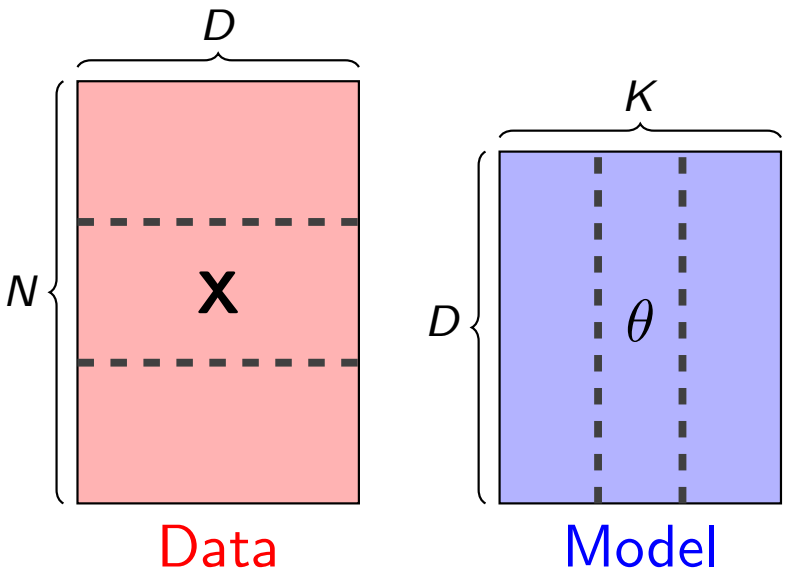
- Model Parallel: Each worker requires requires $\underbrace{\mathcal{O}\left(N \times D\right)}_{\text{bottleneck}} + \mathcal{O}\left(\frac{K \times D}{P}\right)$

Distributed Parameter Estimation

Question

Can we get the best of both worlds?

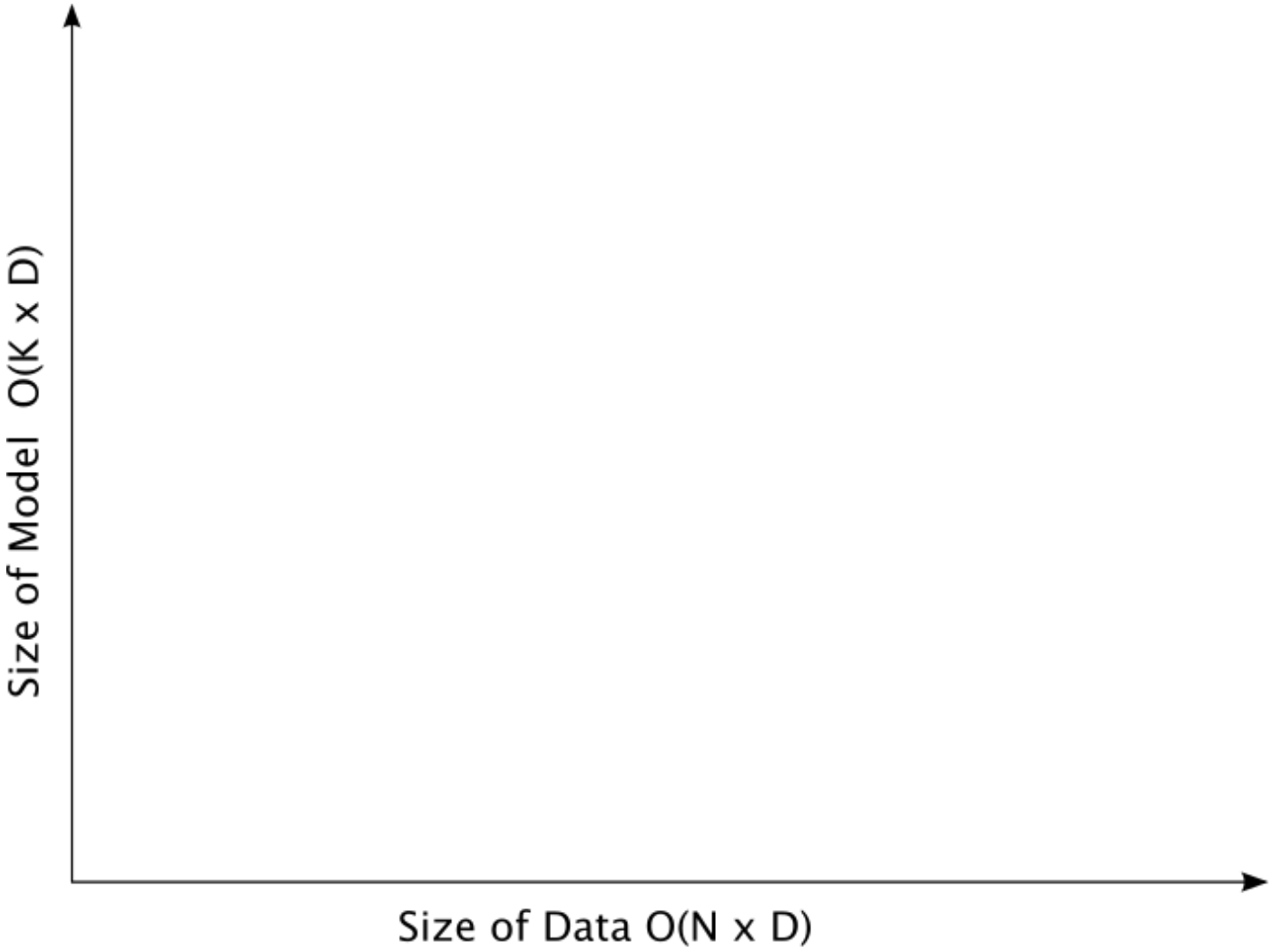
Hybrid Parallelism



Benefits of Hybrid-Parallelism

1. **One versatile method** for all regimes of data and model parallelism

Why Hybrid Parallelism?



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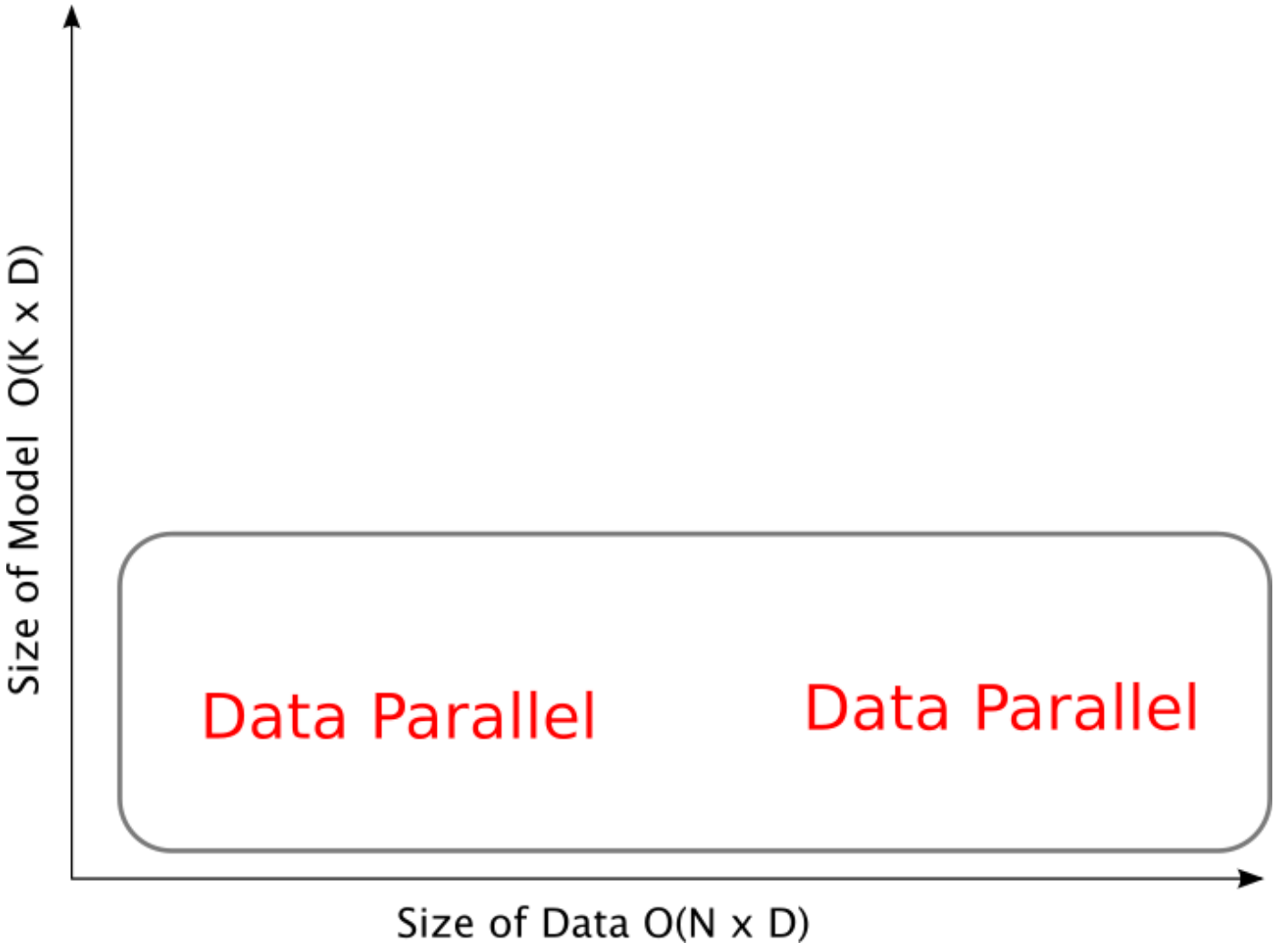

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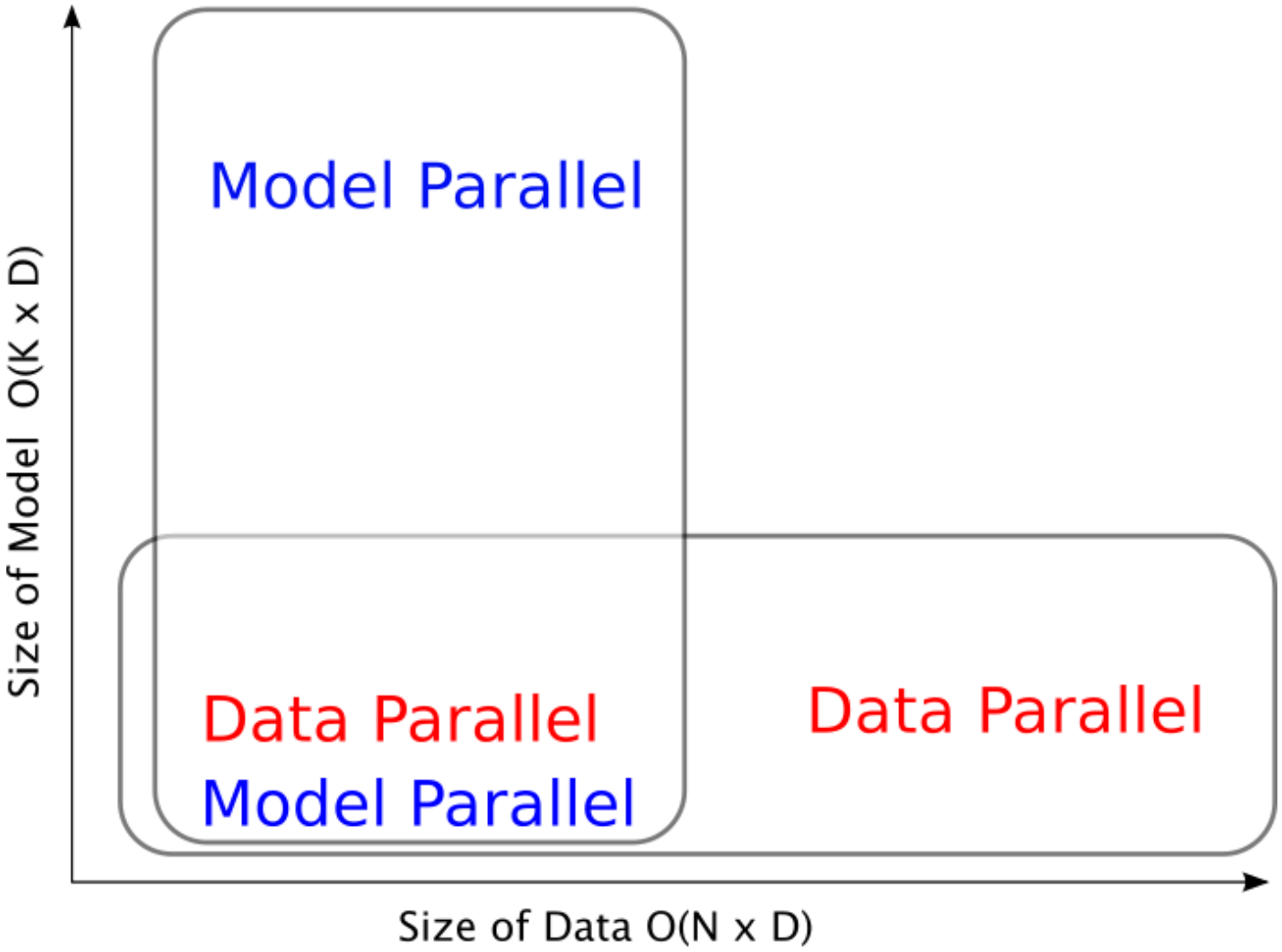

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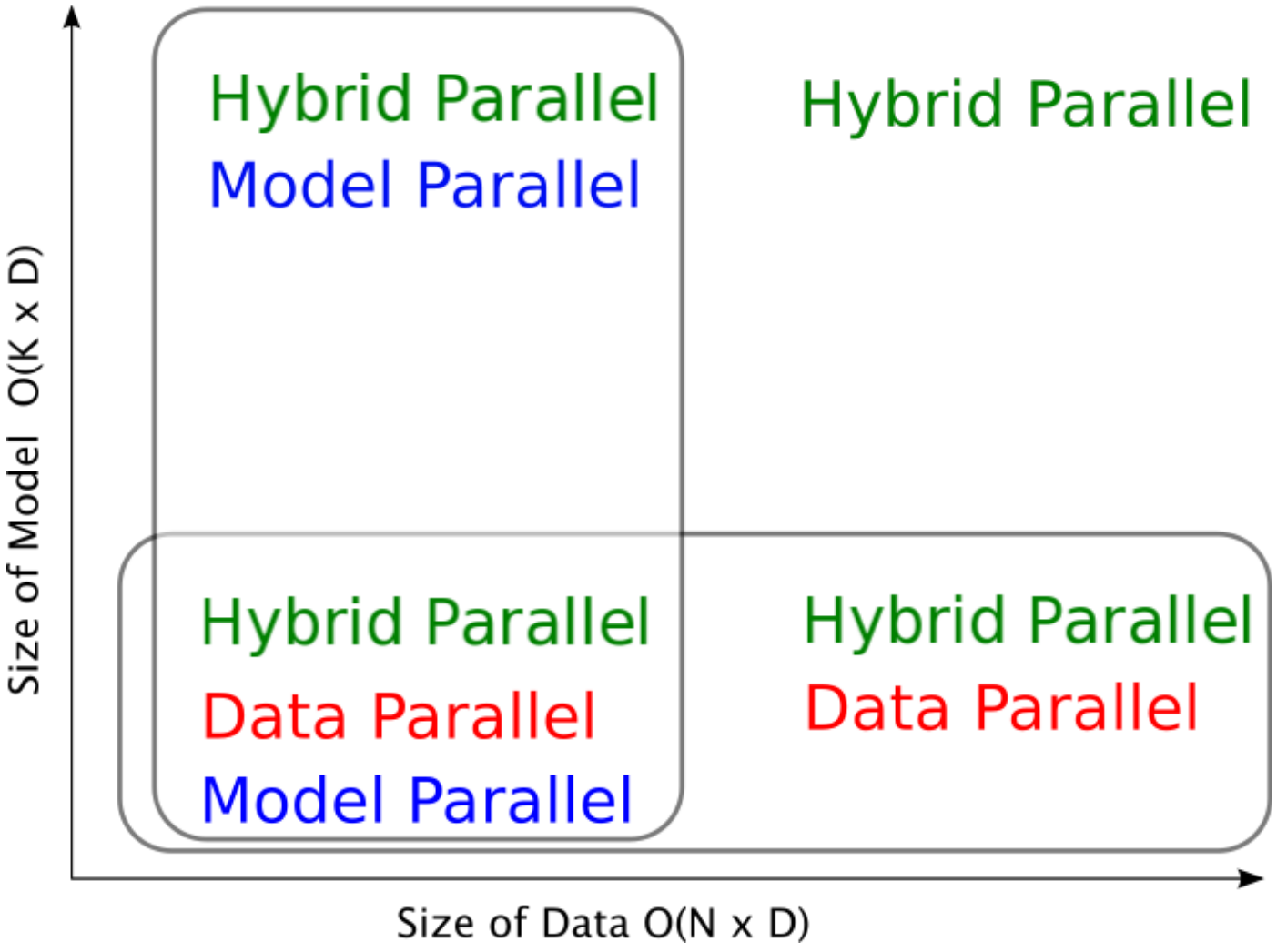

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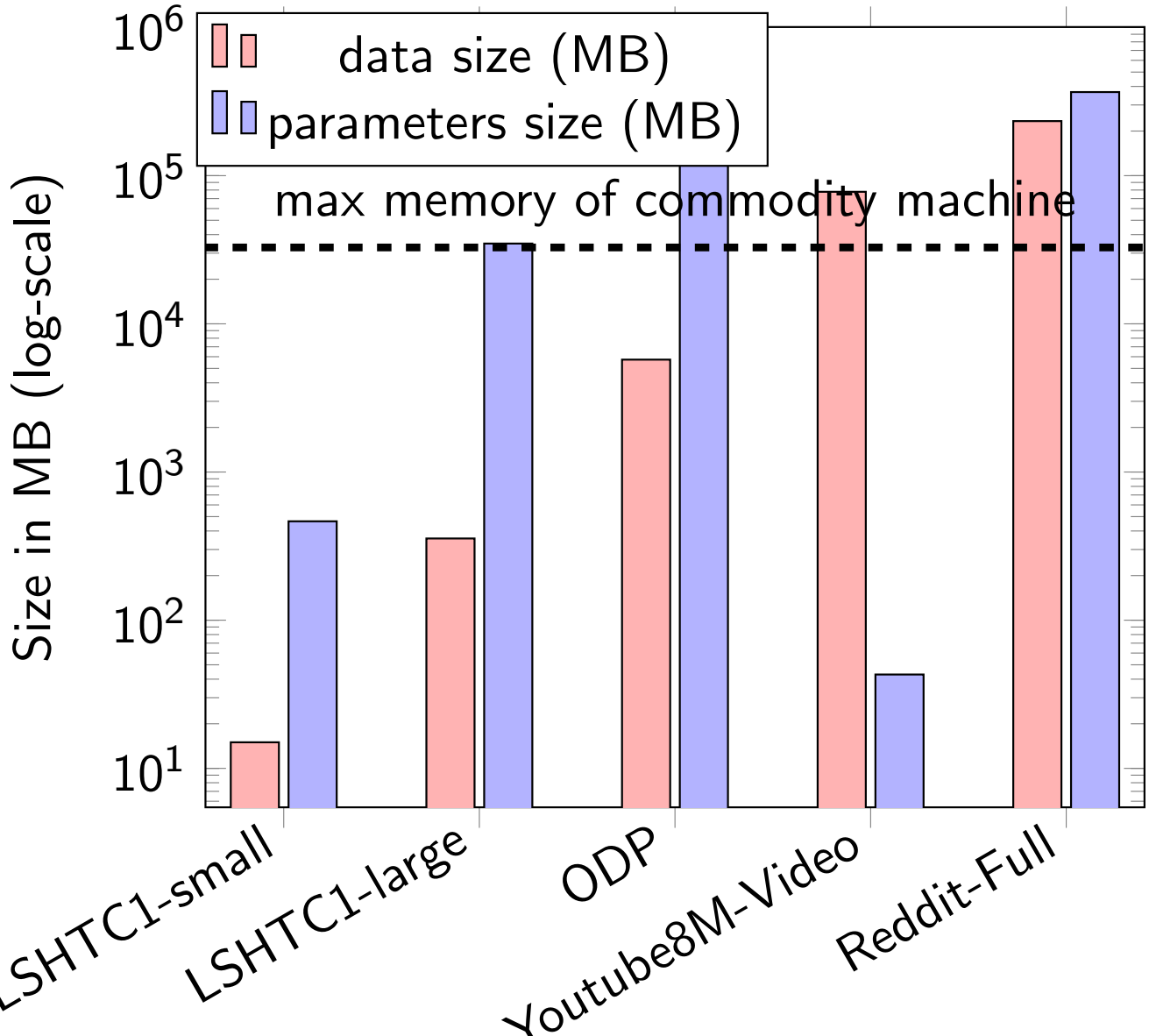

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1. **One versatile method** for all regimes of data and model parallelism
2. **Independent parameter updates** on each worker
3. **Fully de-centralized** and **asynchronous** optimization algorithms

How do we achieve **Hybrid Parallelism** in machine learning models?

Double-Separability

Hybrid Parallelism

Double-Separability

Definition

Let $\{\mathbb{S}_i\}_{i=1}^{m}$ and $\{\mathbb{S}'_j\}_{j=1}^{m'}$ be two families of sets of parameters. A function $f : \prod_{i=1}^{m} \mathbb{S}_i \times \prod_{j=1}^{m'} \mathbb{S}'_j \to \mathbb{R}$ is doubly separable if $\exists\ f_{ij} : \mathbb{S}_i \times \mathbb{S}'_j \to \mathbb{R}$ for each $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, m'$ such that:

$$f(\theta_1, \theta_2, \ldots, \theta_m, \theta'_1, \theta'_2, \ldots, \theta'_{m'}) = \sum_{i=1}^{m} \sum_{j=1}^{m'} f_{ij}(\theta_i, \theta'_j)$$

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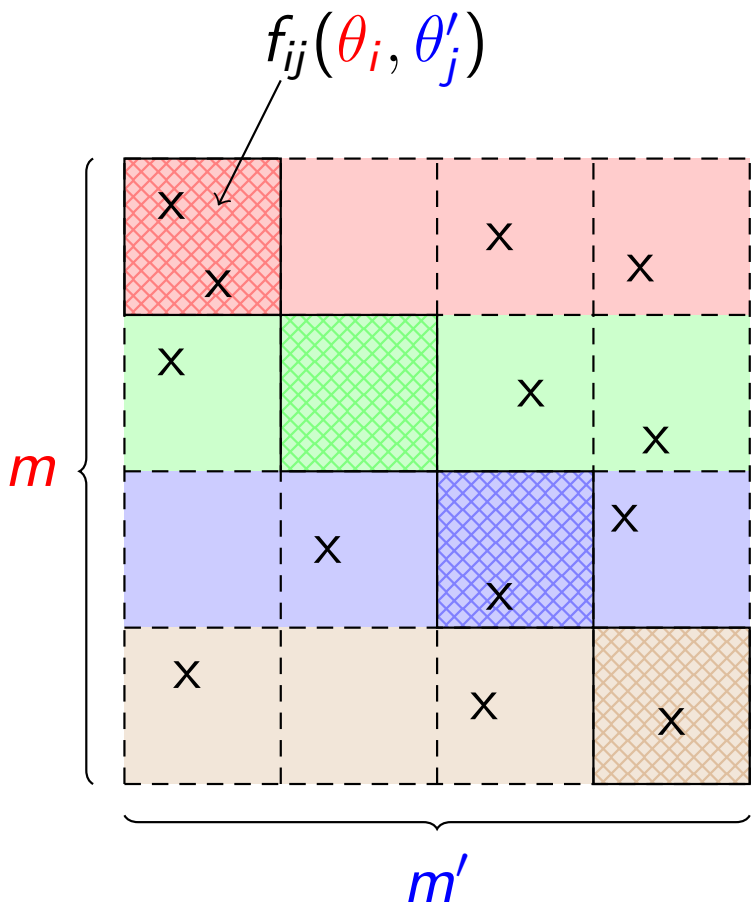

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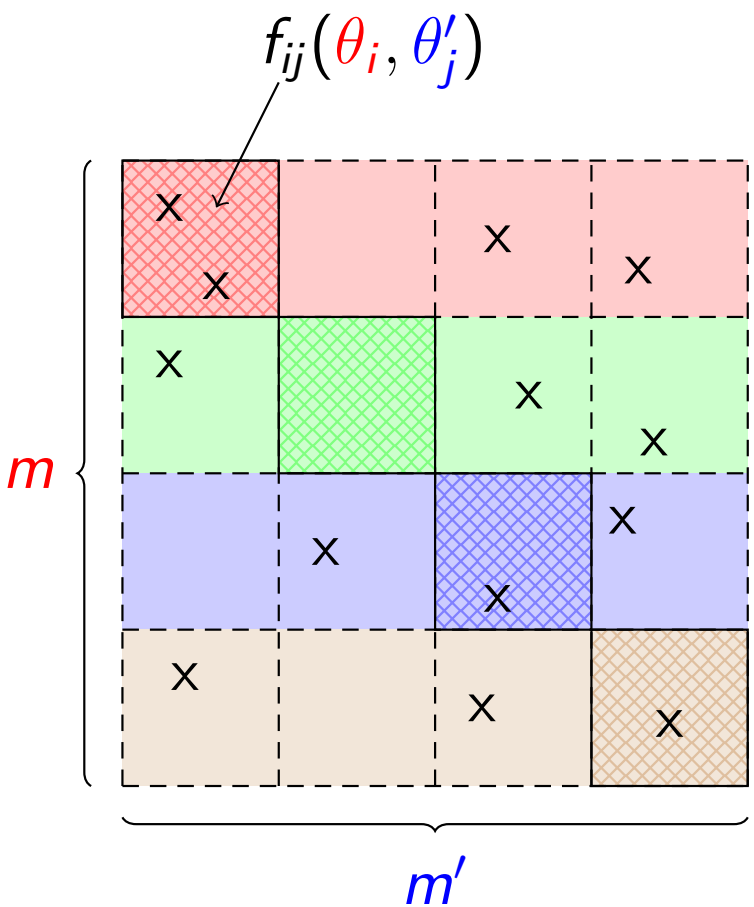
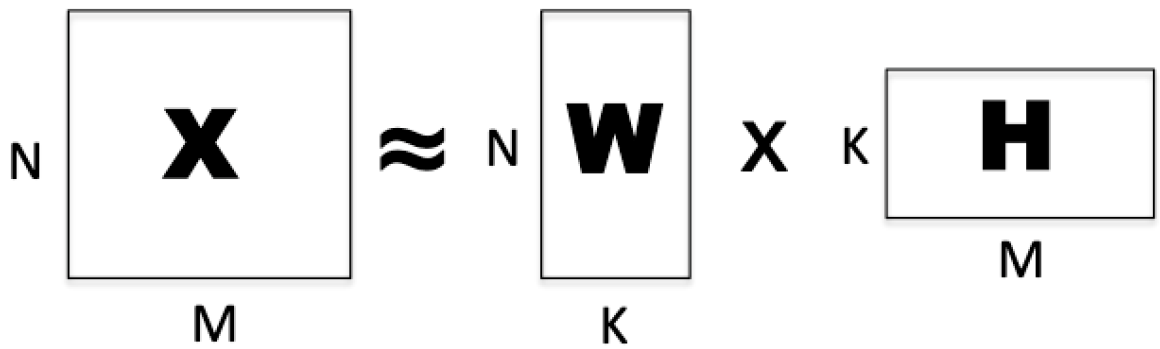


f_{ij} corresponding to highlighted diagonal blocks can be computed **independently** and in **parallel**

Direct Double-Separability

e.g. Matrix Factorization



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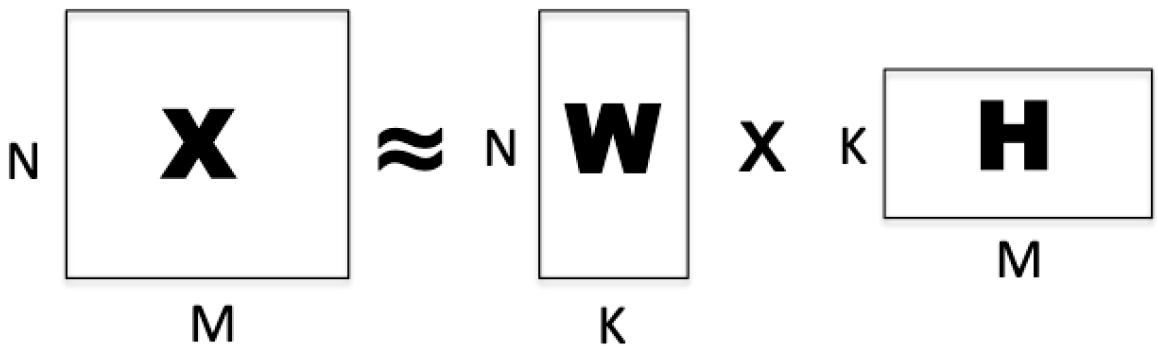

$$\mathcal{L}(\mathbf{w}_1, \mathbf{w}_2, \ldots, \mathbf{w}_N, \mathbf{h}_1, \mathbf{h}_2, \ldots, \mathbf{h}_M) = \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{M} (X_{ij} - \langle \mathbf{w}_i, \mathbf{h}_j \rangle)^2$$

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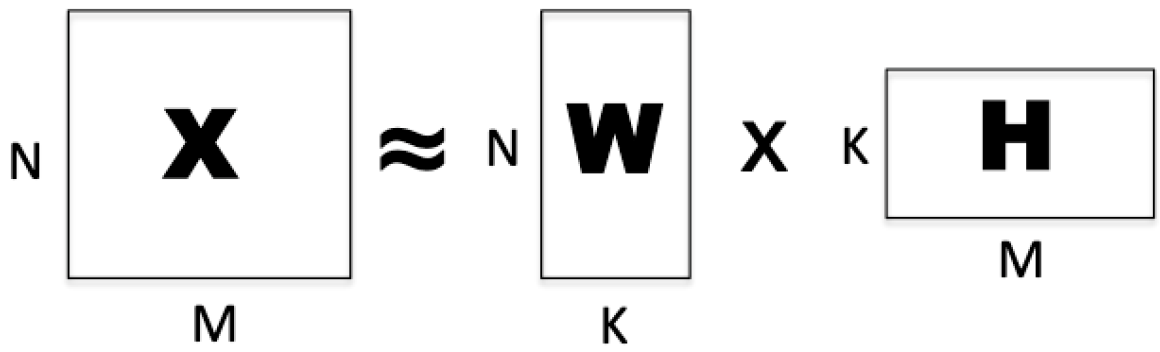

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Objective function is trivially doubly-separable! [Yun et al 2014]

Others require Reformulations

original objective $f(\cdot)$

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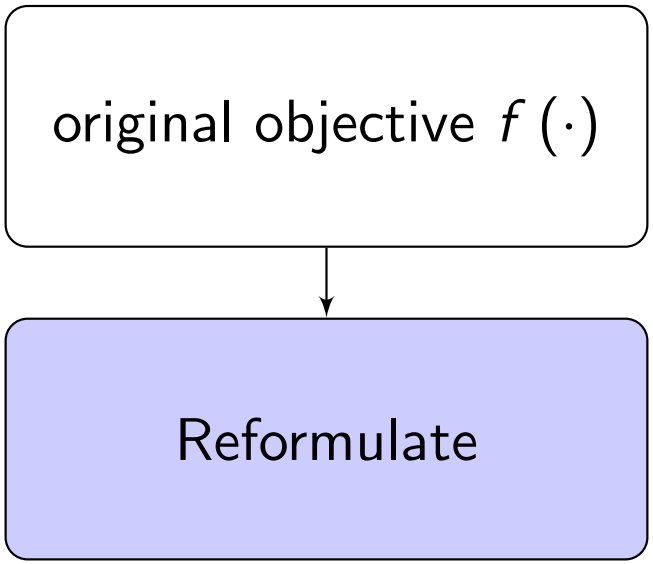

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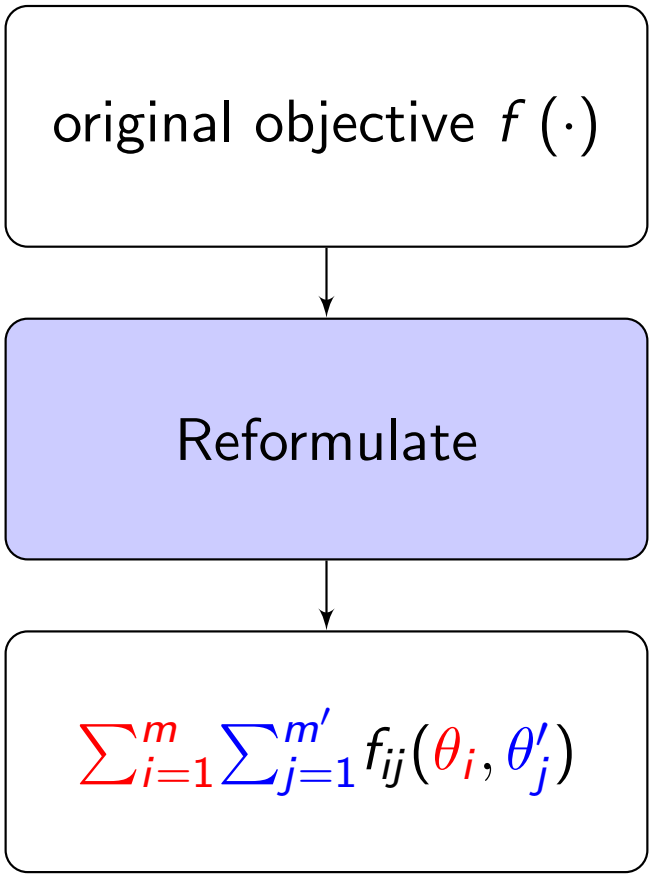

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Study frequentist and bayesian models which **do not have direct double-separability** in their parameter estimation,

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2. Design asynchronous distributed optimization algorithms

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Multinomial Logistic Regression (MLR)

Given

- Training data $(\mathbf{x}_i, y_i)_{i=1,\ldots,N}$ where $\mathbf{x}_i \in \mathbb{R}^D$
- corresp. labels $y_i \in \{1, 2, \ldots, K\}$
- N, D and K are large ($N >>> D >> K$)

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Goal

The probability that $\mathbf{x}_i$ belongs to class k is given by:

$$p(y_i = k|\mathbf{x}_i, W) = \frac{\exp(\mathbf{w}_k{}^T \mathbf{x}_i)}{\sum_{j=1}^{K} \exp(\mathbf{w}_j{}^T \mathbf{x}_i)}$$

where $W = \{\mathbf{w}_1, \mathbf{w}_2, \ldots, \mathbf{w}_K\}$ denotes the parameter for the model.

Multinomial Logistic Regression (MLR)

The corresponding l_2 **regularized negative log-likelihood loss**:

$$\min_{W} \frac{\lambda}{2} \sum_{k=1}^{K} \|\mathbf{w}_k\|^2 - \frac{1}{N} \sum_{i=1}^{N} \sum_{k=1}^{K} y_{ik} \mathbf{w}_k{}^T \mathbf{x}_i + \frac{1}{N} \sum_{i=1}^{N} \log \left(\sum_{k=1}^{K} \exp(\mathbf{w}_k{}^T \mathbf{x}_i) \right)$$

where λ is the regularization hyper-parameter.

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where λ is the regularization hyper-parameter.

Reformulation into Doubly-Separable form

Log-concavity bound [GopYan13]

$$\log(\gamma) \leq a \cdot \gamma - \log(a) - 1, \quad \forall \gamma, a > 0,$$

where a is a variational parameter. This bound is tight when $a = \frac{1}{\gamma}$.

Reformulating the objective of MLR

$$\min_{W,A} \frac{\lambda}{2} \sum_{k=1}^{K} \|\mathbf{w}_k\|^2 + \frac{1}{N} \sum_{i=1}^{N} \left(- \sum_{k=1}^{K} y_{ik} \mathbf{w}_k{}^T \mathbf{x}_i + a_i \sum_{k=1}^{K} \exp(\mathbf{w}_k{}^T \mathbf{x}_i) - \log(a_i) - 1 \right)$$

where a_i can be computed in closed form as:

$$a_i = \frac{1}{\sum_{k=1}^{K} \exp(\mathbf{w}_k{}^T \mathbf{x}_i)}$$

Doubly-Separable Multinomial Logistic Regression (DS-MLR)

Doubly-Separable form

$$\min_{W,A} \sum_{i=1}^{N}\sum_{k=1}^{K}\left(\frac{\lambda\|\mathbf{w}_k\|^2}{2N} - \frac{y_{ik}\mathbf{w}_k{}^T\mathbf{x}_i}{N} - \frac{\log a_i}{NK} + \frac{\exp(\mathbf{w}_k{}^T\mathbf{x}_i + \log a_i)}{N} - \frac{1}{NK}\right)$$

Doubly-Separable Multinomial Logistic Regression (DS-MLR)

Stochastic Gradient Updates

Each term in stochastic update depends on only data point i and class k.

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- $\mathbf{w}_k{}^{t+1} \leftarrow \mathbf{w}_k{}^{t} - \eta_t K \left(\lambda \mathbf{x}_i - y_{ik}\mathbf{x}_i + \exp\left(\mathbf{w}_k{}^{T}\mathbf{x}_i + \log a_i\right)\mathbf{x}_i\right)$

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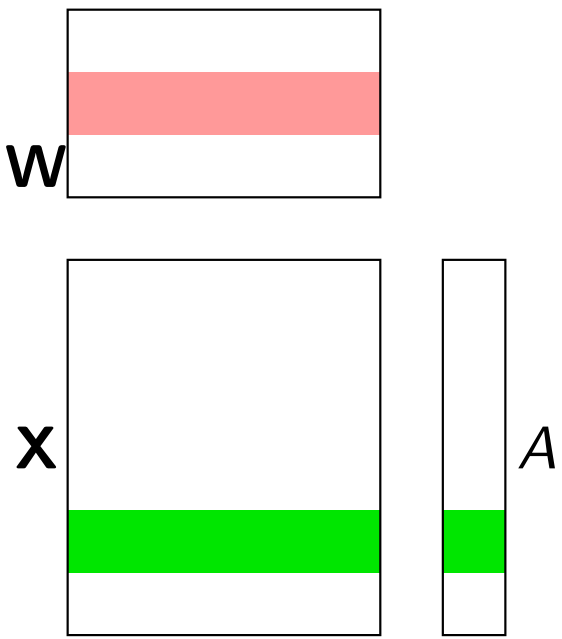
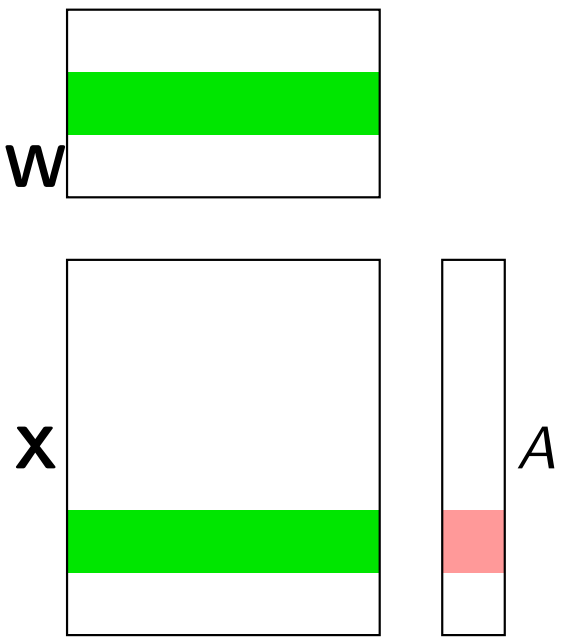
- $\log a_i{}^{t+1} \leftarrow \log a_i{}^{t} - \eta_t K \left(\exp(\mathbf{w}_k{}^{T} \mathbf{x}_i + \log a_i) - \frac{1}{K} \right)$

Access Pattern of updates: Stoch $\mathbf{w}_k$, Stoch a_i



(a) Updating $\mathbf{w}_k$ only requires computing a_i



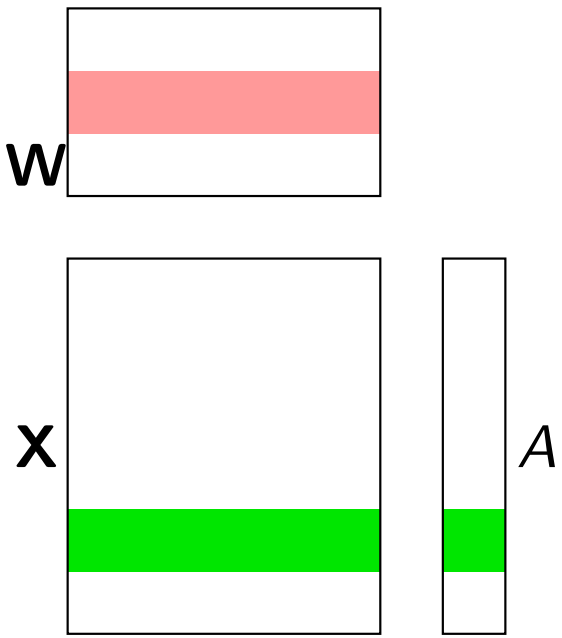
(b) Updating a_i only requires accessing $\mathbf{w}_k$ and $\mathbf{x}_i$.

Updating a_i: Closed form instead of Stoch update

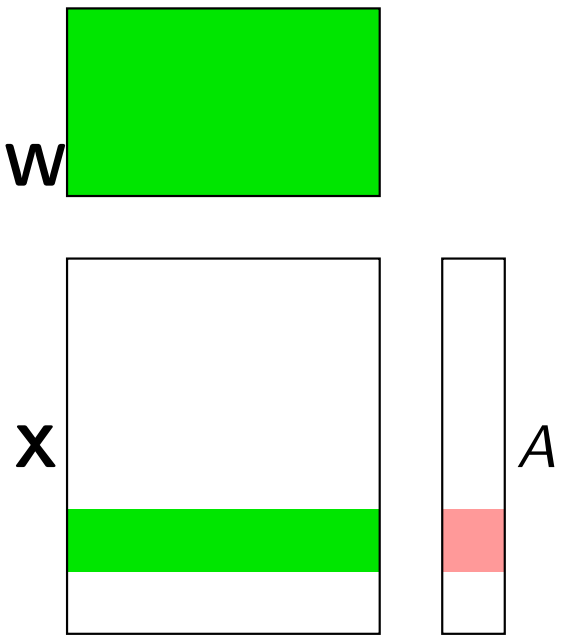
Closed-form update for a_i

$$a_i = \frac{1}{\sum_{k=1}^{K} \exp(\mathbf{w}_k{}^T \mathbf{x}_i)}$$

Access Pattern of updates: Stoch $\mathbf{w}_k$, Exact a_i



(a) Updating $\mathbf{w}_k$ only requires computing a_i



(b) Updating a_i requires accessing entire $\mathbf{W}$. Synchronization bottleneck!

Updating a_i: Avoiding bulk-synchronization

Closed-form update for a_i

$$a_i = \frac{1}{\sum_{k=1}^{K} \exp(\mathbf{w}_k{}^T \mathbf{x}_i)}$$

Updating a_i: Avoiding bulk-synchronization

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$$a_i = \frac{1}{\sum_{k=1}^{K} \exp(\mathbf{w}_k{}^T \mathbf{x}_i)}$$

- Each worker computes partial sum using the $\mathbf{w}_k$ it owns.

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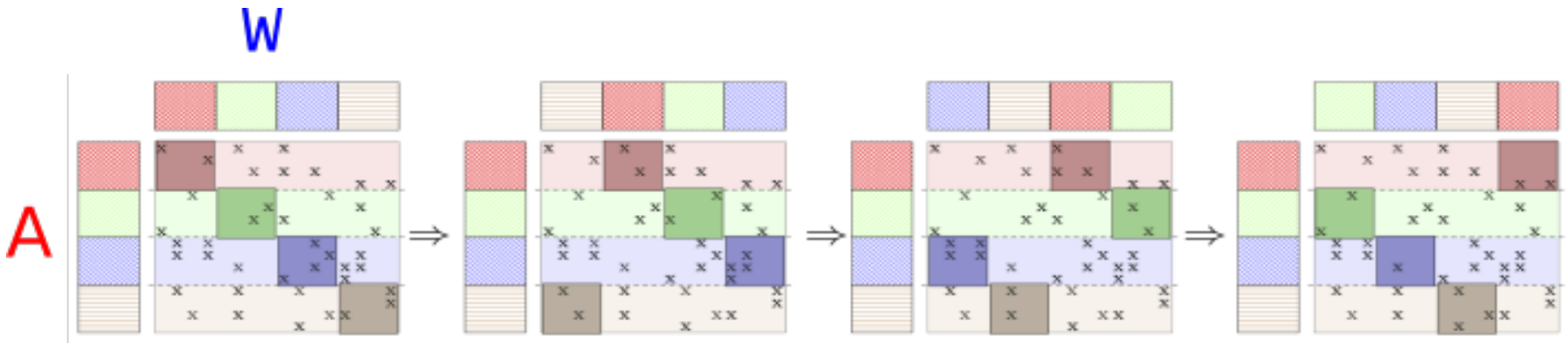
Closed-form update for a_i

$$a_i = \frac{1}{\sum_{k=1}^{K} \exp(\mathbf{w}_k{}^T \mathbf{x}_i)}$$

- Each worker computes partial sum using the $\mathbf{w}_k$ it owns.
- P workers: After P rounds the global sum is available

Parallelization: Synchronous

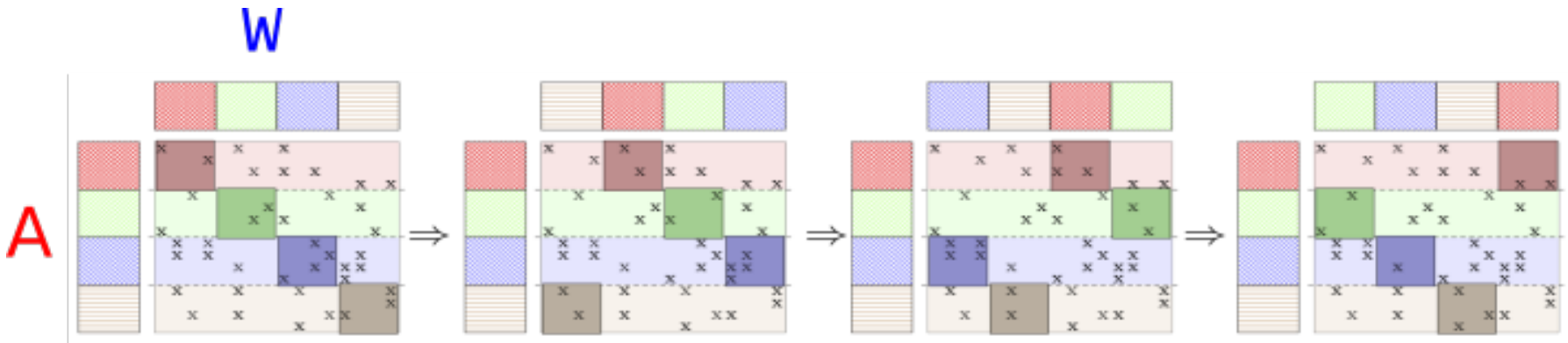
DSGD [Gemulla et al., 2011]



- **X** and local parameters **A** are partitioned horizontally $(1, \ldots, N)$

Parallelization: Synchronous

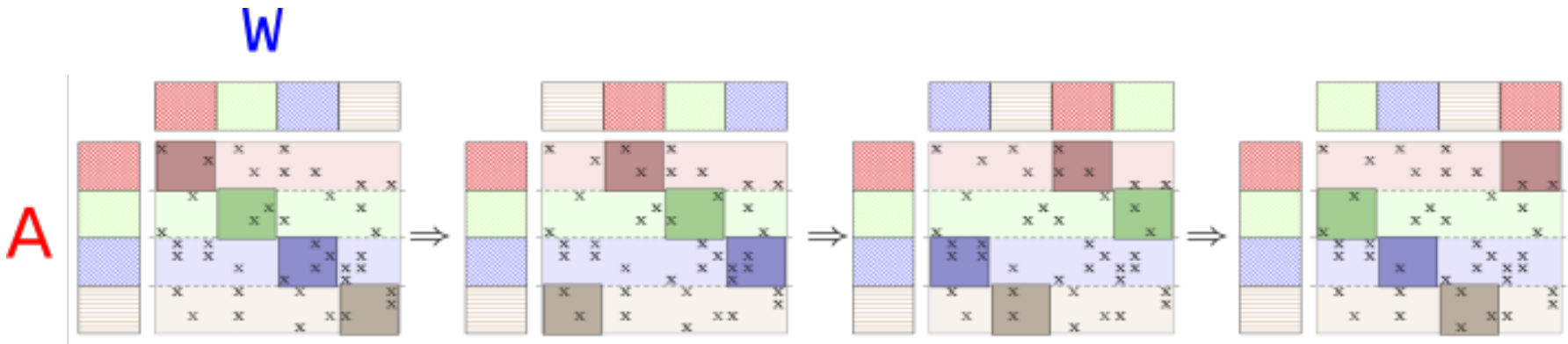
DSGD [Gemulla et al., 2011]



- **X** and local parameters **A** are partitioned horizontally $(1, \ldots, N)$
- Global model parameters **W** are partitioned vertically $(1, \ldots, K)$

Parallelization: Synchronous

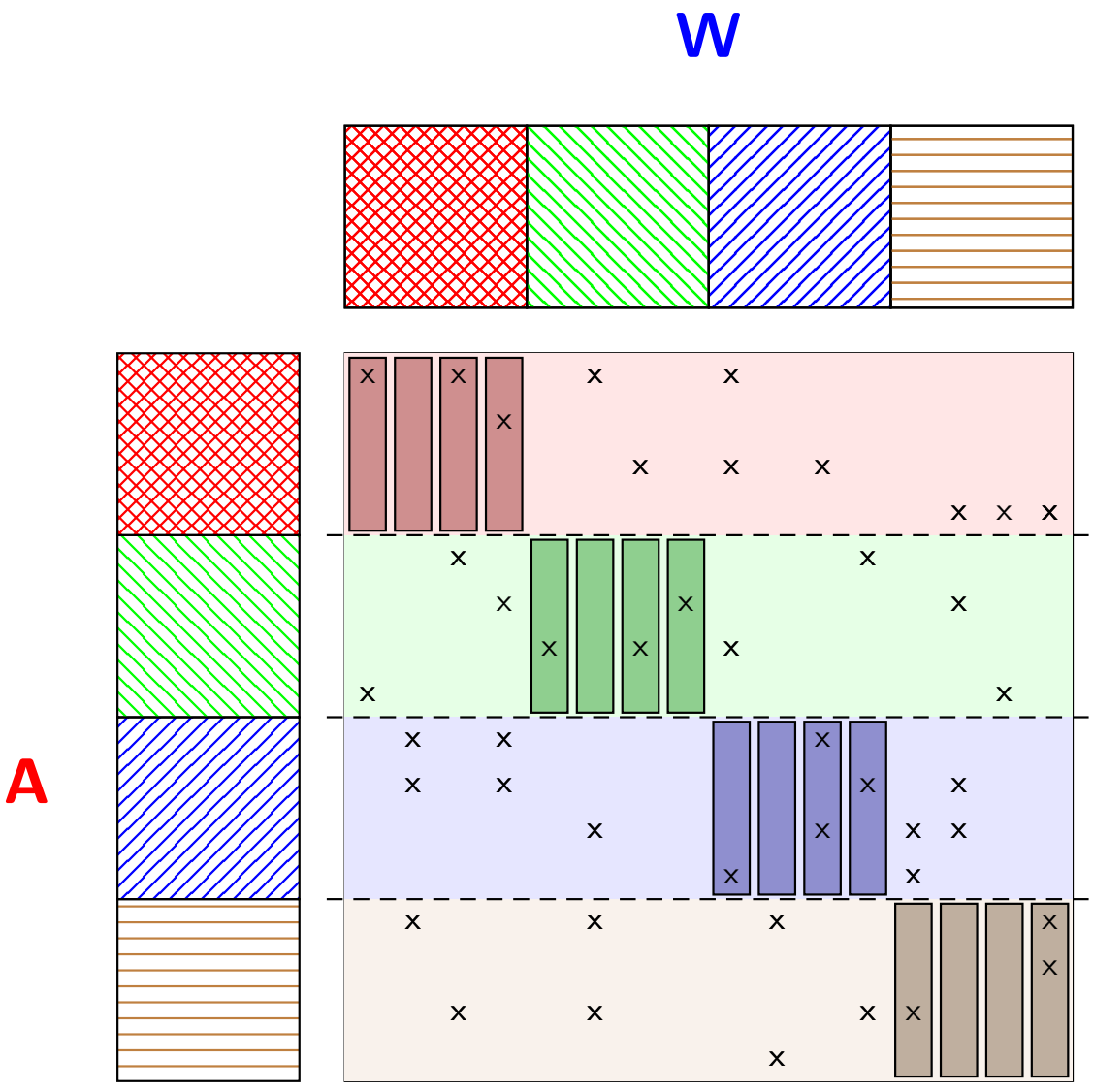
DSGD [Gemulla et al., 2011]



- **X** and local parameters **A** are partitioned horizontally $(1, \ldots, N)$
- Global model parameters **W** are partitioned vertically $(1, \ldots, K)$
- $P = 4$ workers work on mutually-exclusive blocks of **A** and **W**

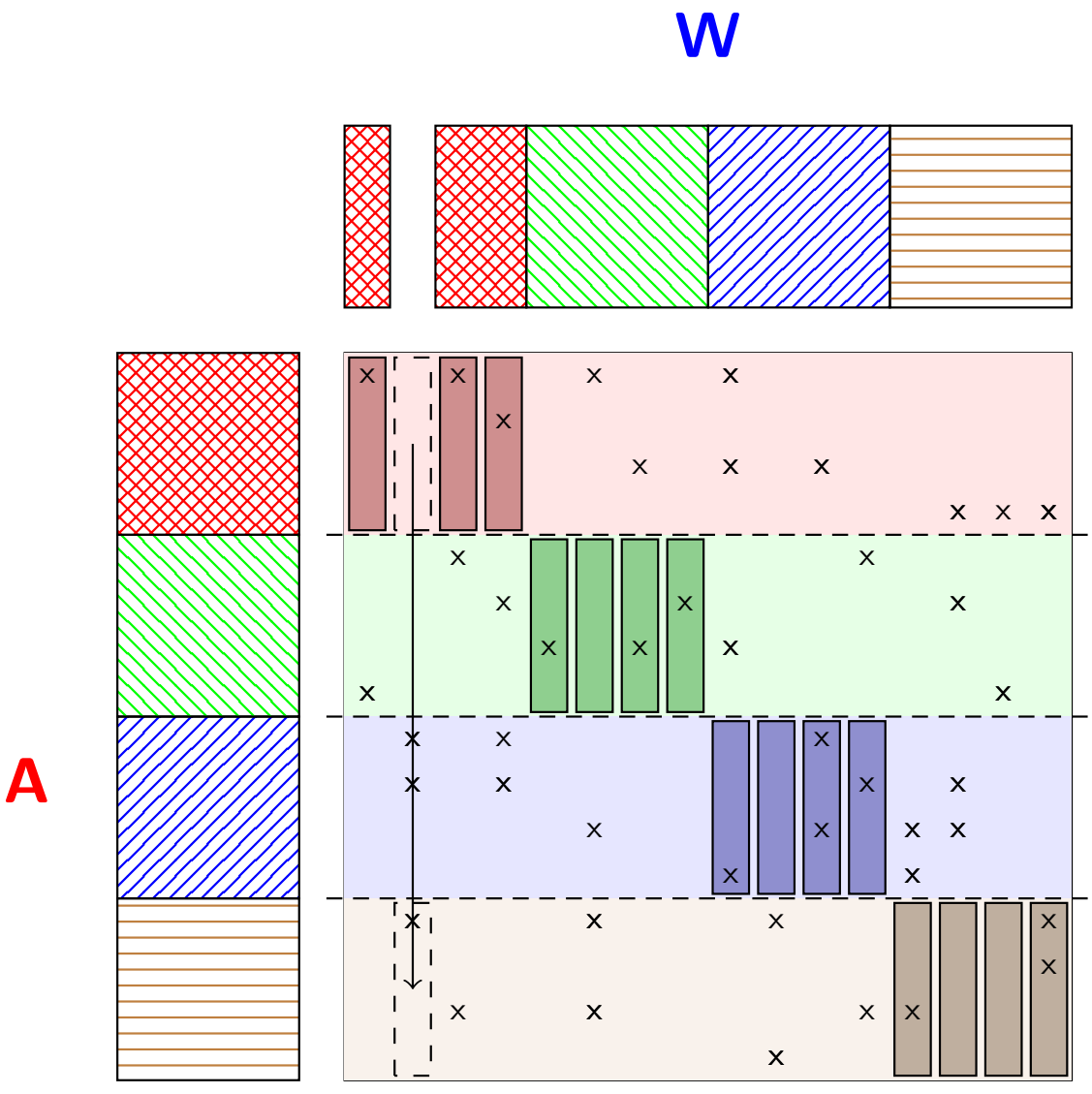
Parallelization: Asynchronous

NOMAD [Yun et al., 2014]



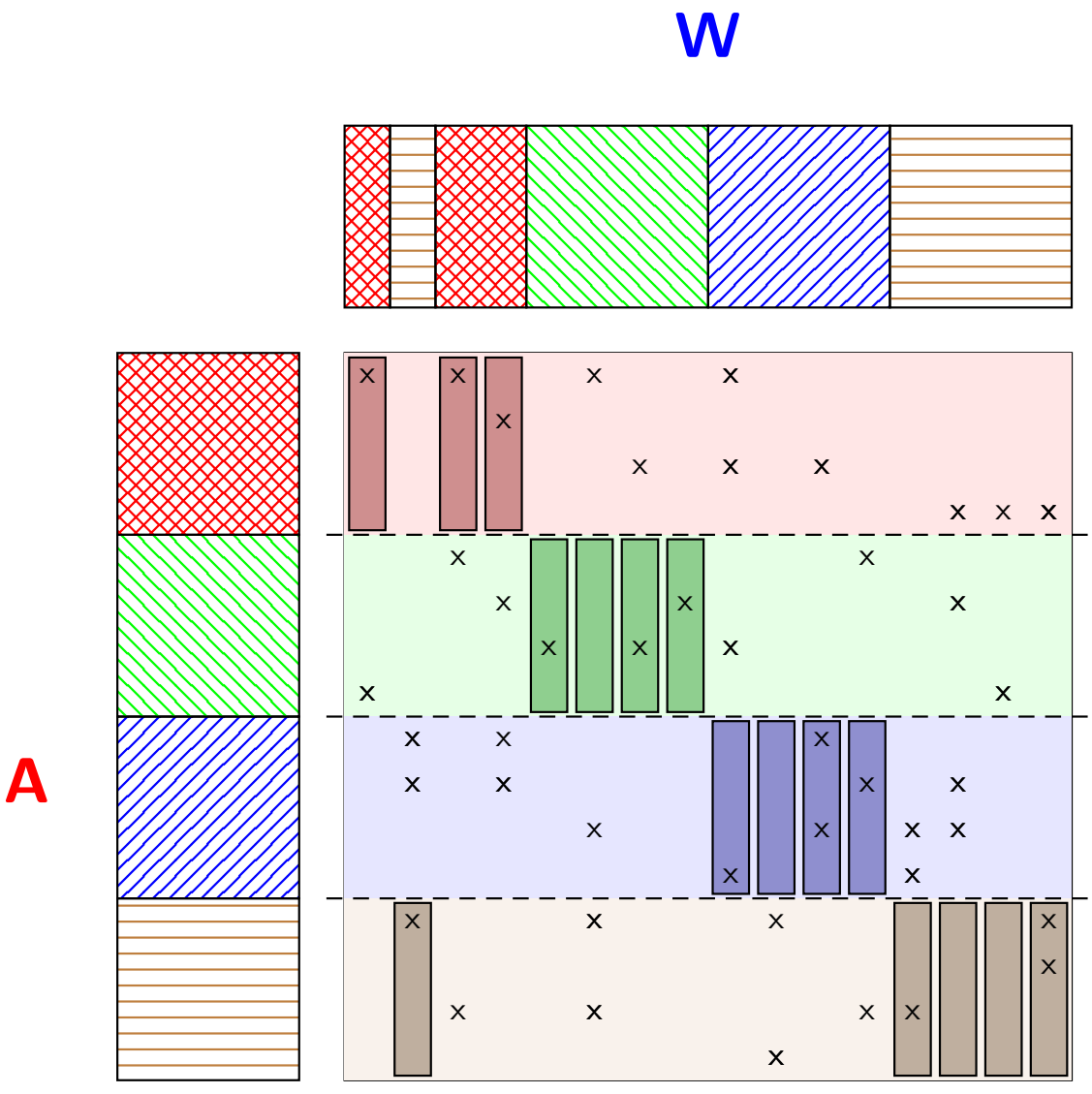
Parallelization: Asynchronous

NOMAD [Yun et al 2014]



Parallelization: Asynchronous

NOMAD [Yun et al 2014]



Parallelization: Asynchronous

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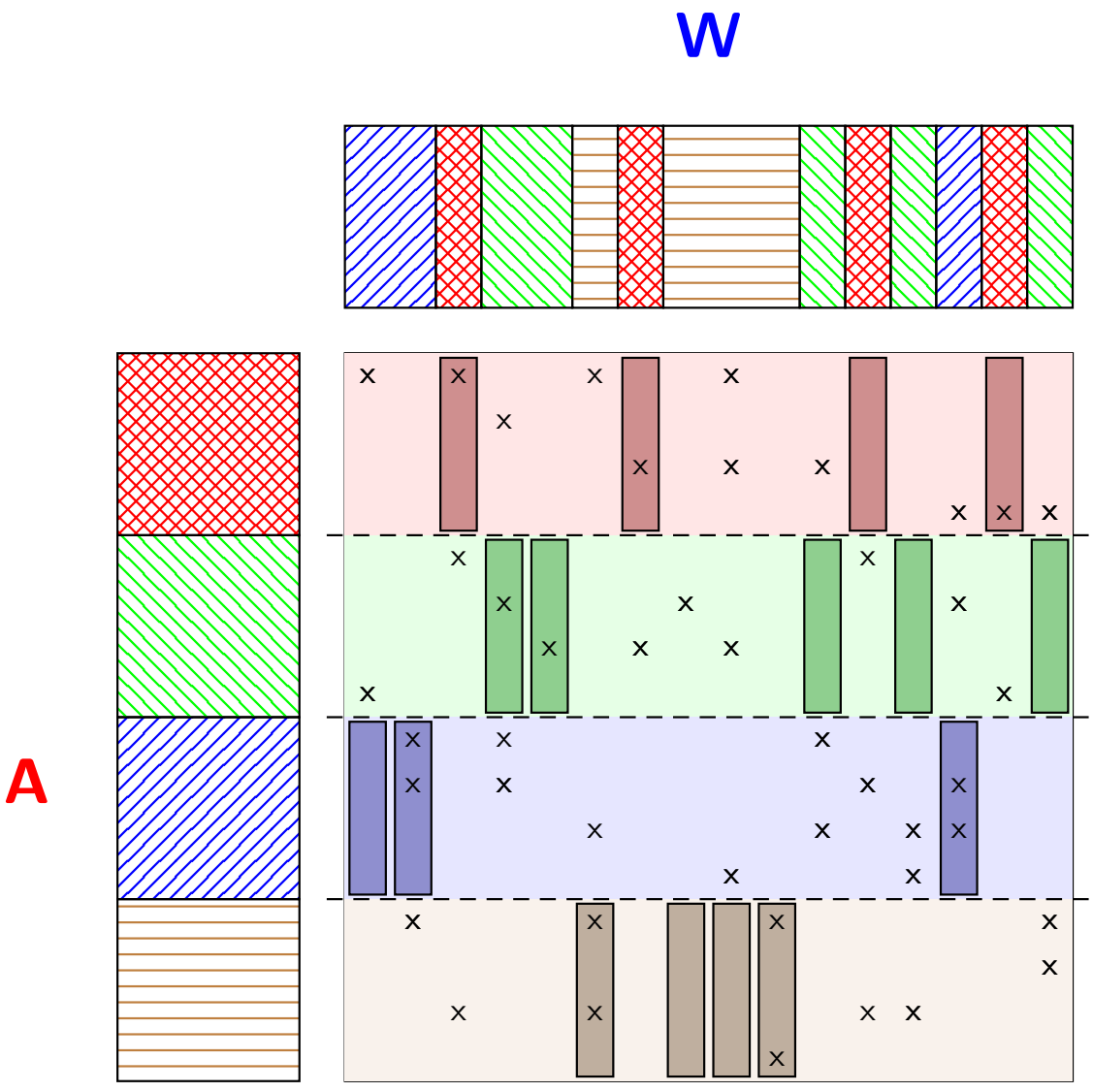
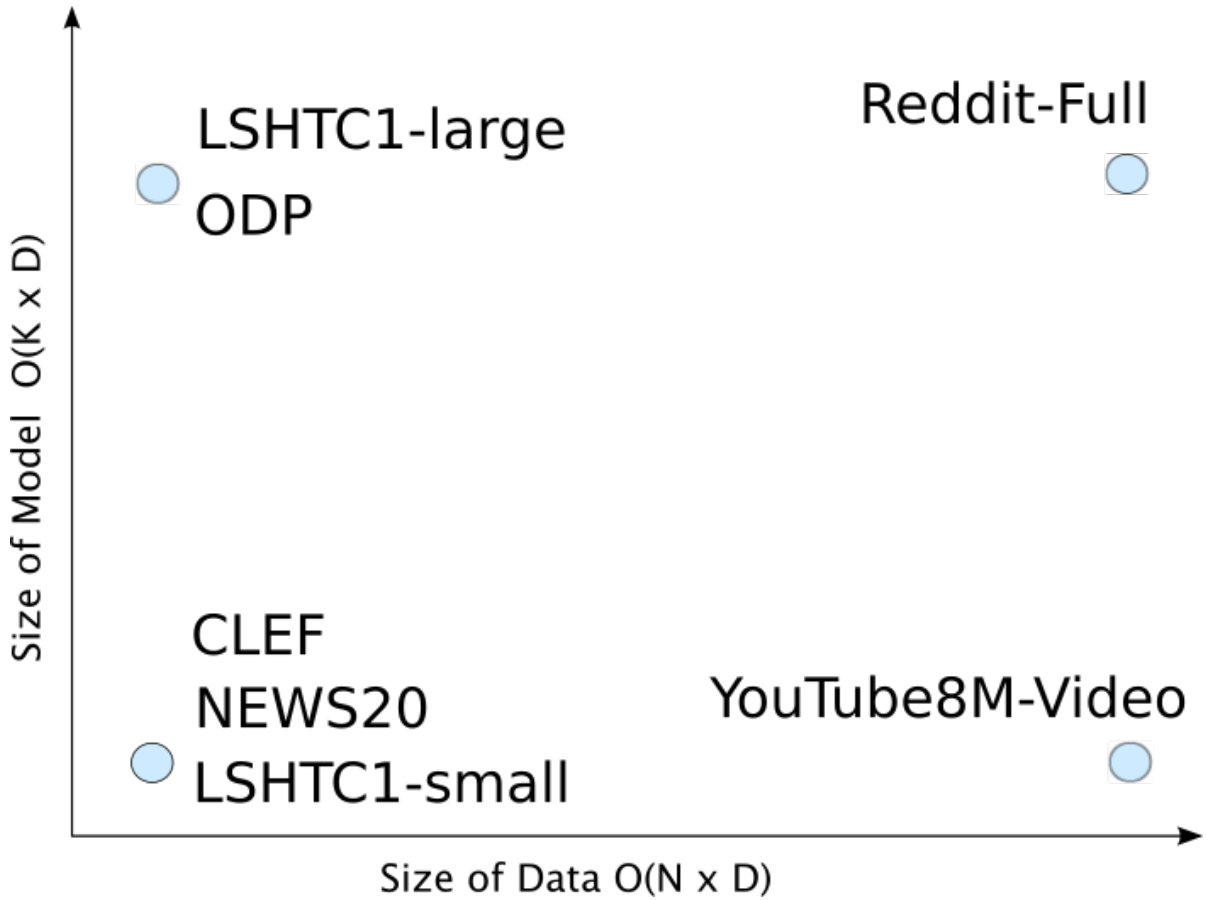


Experiments: Datasets

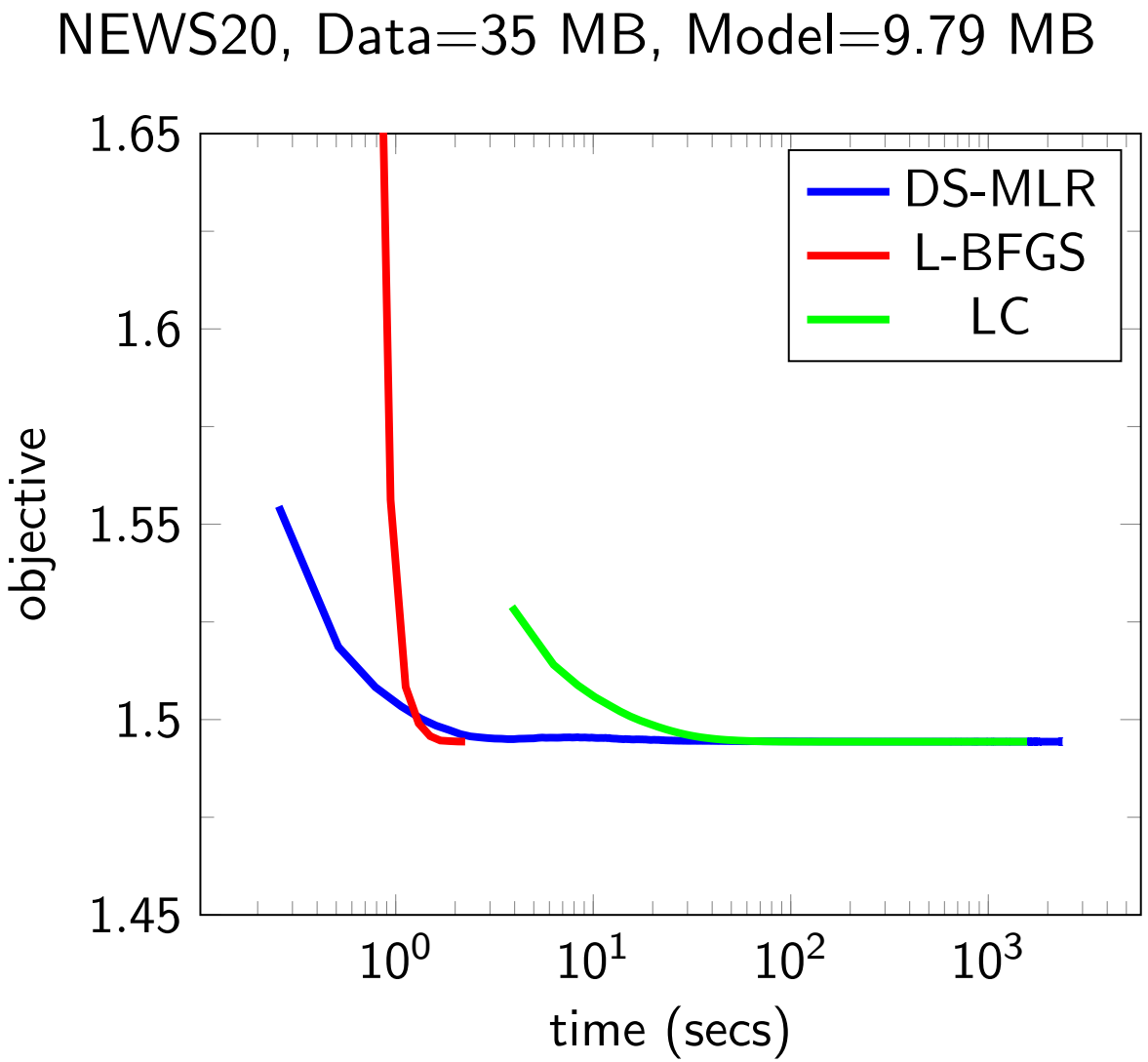


Experiments: Datasets

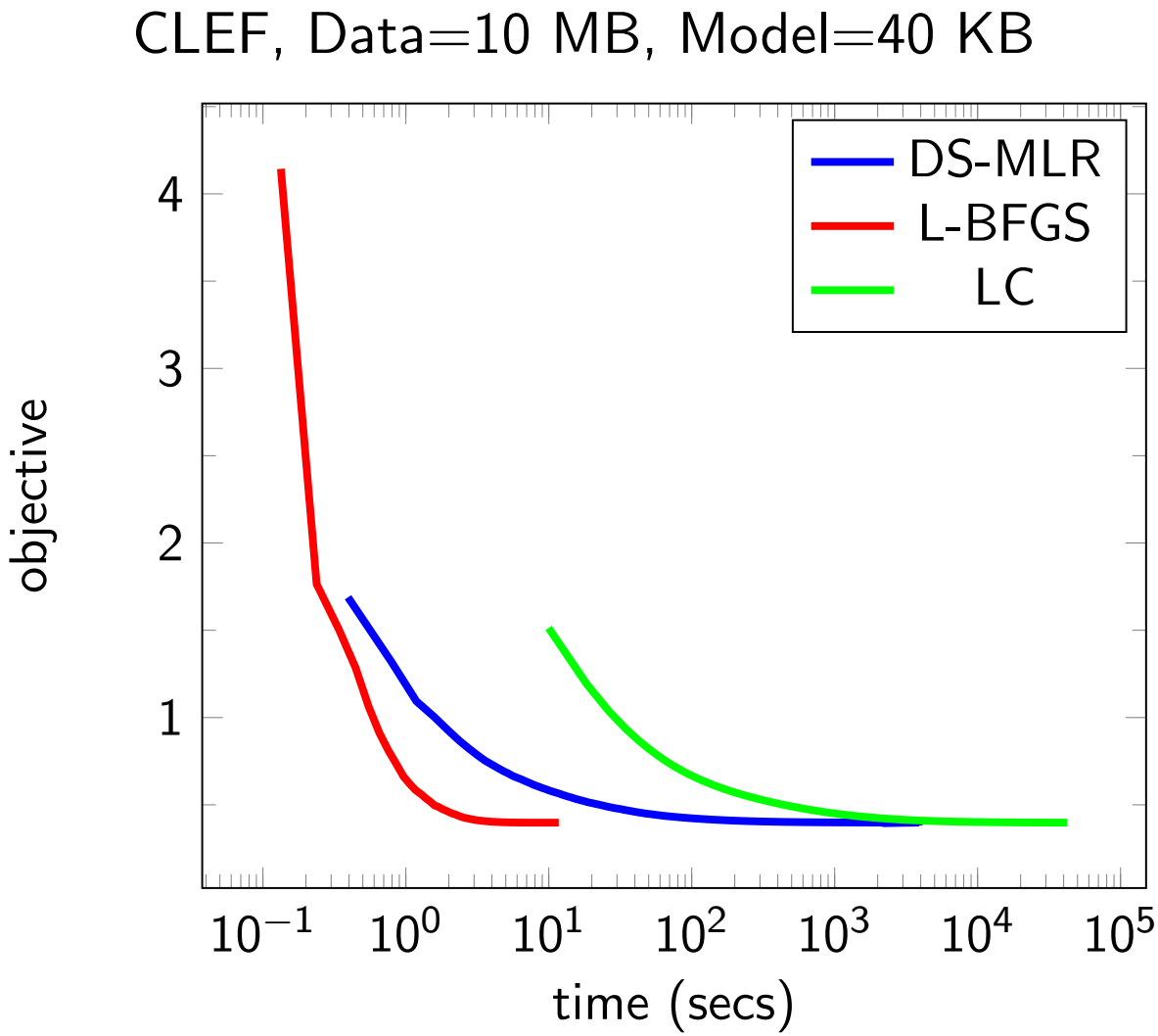
Dataset	# instances	# features	#classes	data (train + test)	parameters	sparsity (% nnz)
CLEF	10,000	80	63	9.6 MB + 988 KB	40 KB	100
NEWS20	11,260	53,975	20	21 MB + 14 MB	9.79 MB	0.21
LSHTC1-small	4,463	51,033	1,139	11 MB + 4 MB	465 MB	0.29
LSHTC1-large	93,805	347,256	12,294	258 MB + 98 MB	34 GB	0.049
ODP	1,084,404	422,712	105,034	3.8 GB + 1.8 GB	355 GB	0.0533
YouTube8M-Video	4,902,565	1,152	4,716	59 GB + 17 GB	43 MB	100
Reddit-Full	211,532,359	1,348,182	33,225	159 GB + 69 GB	358 GB	0.0036

Table: **Characteristics of the datasets used**

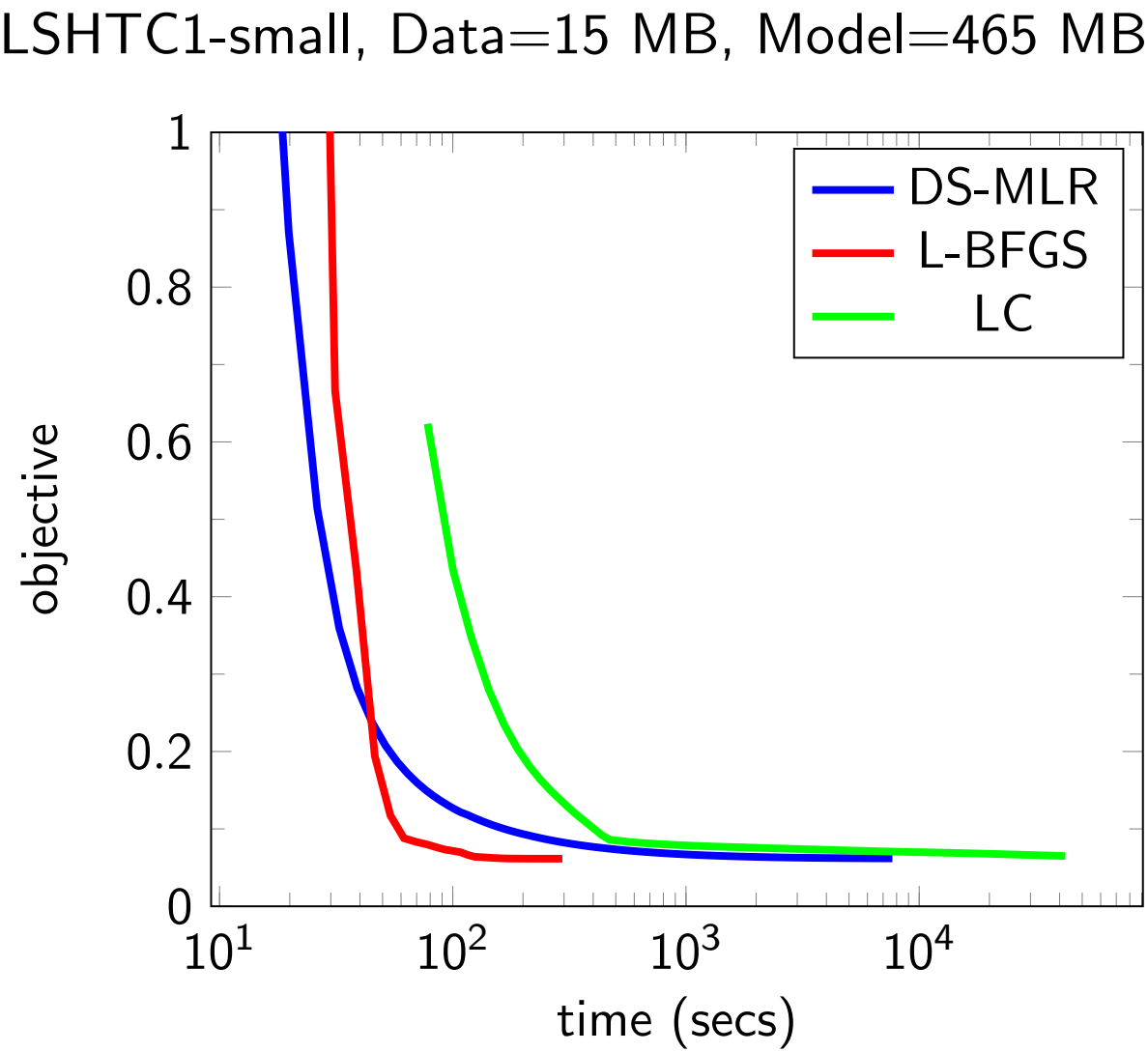
Experiments: Single Machine



Experiments: Single Machine

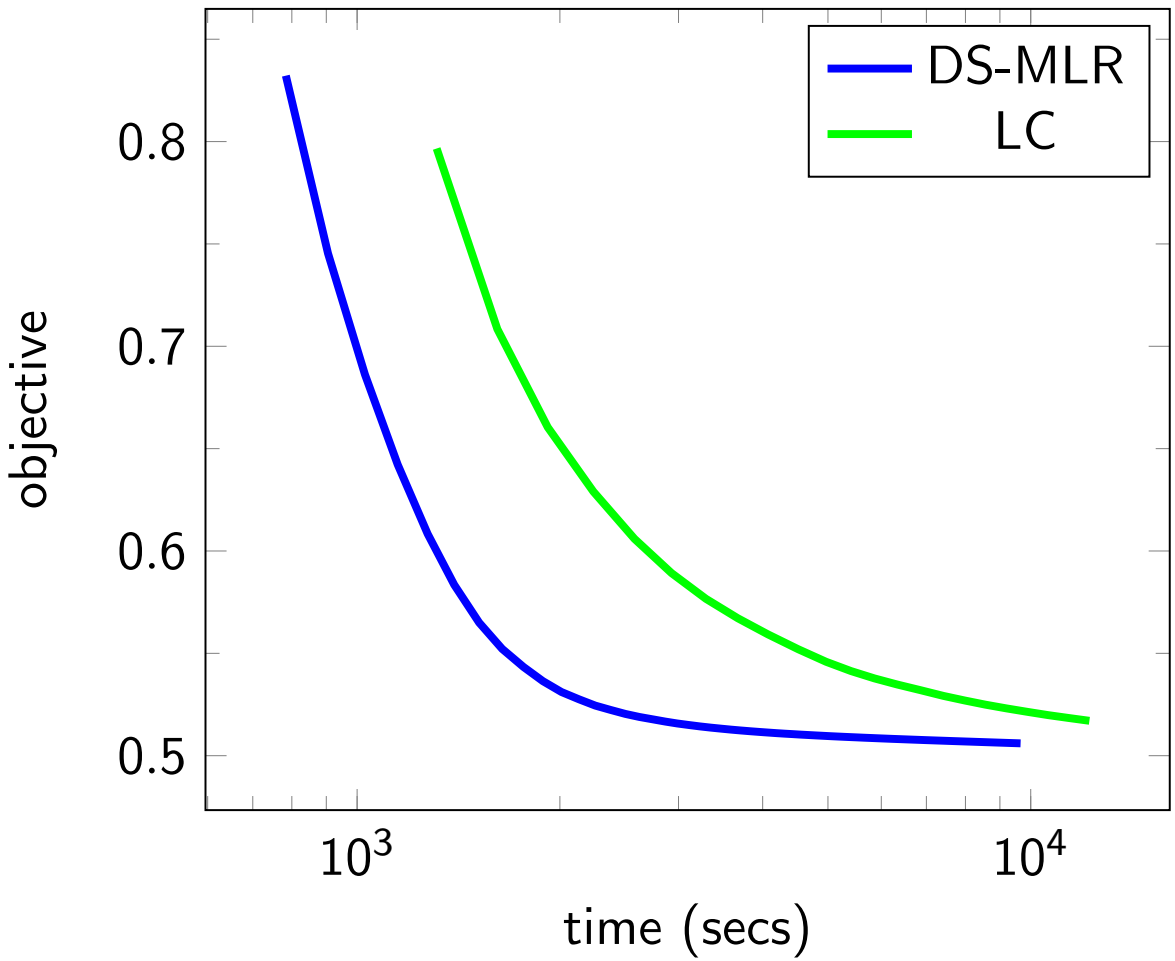


Experiments: Single Machine

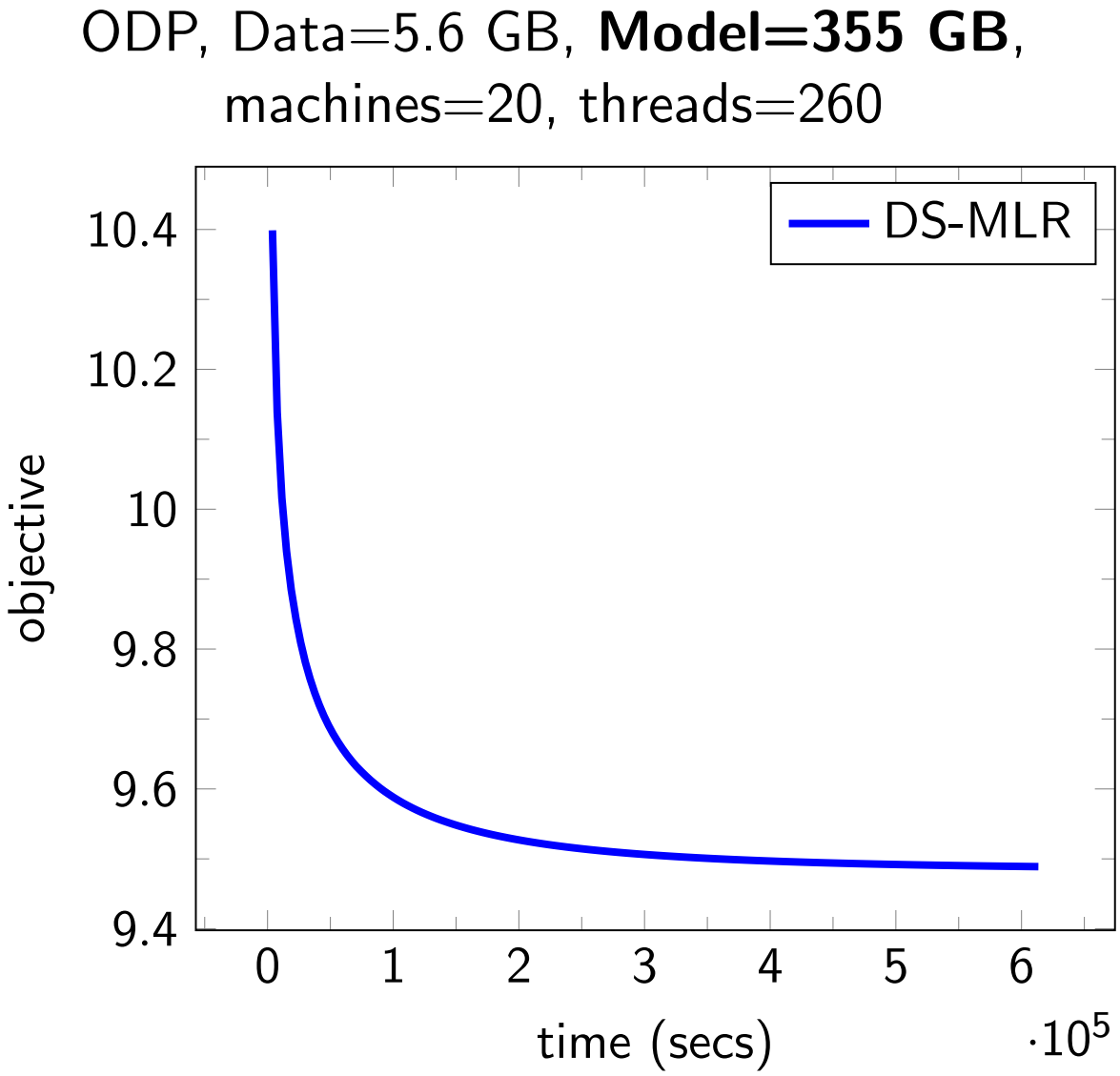


Experiments: Multi Machine

LSHTC1-large, Data=356 MB, **Model=34 GB**,
machines=4, threads=12

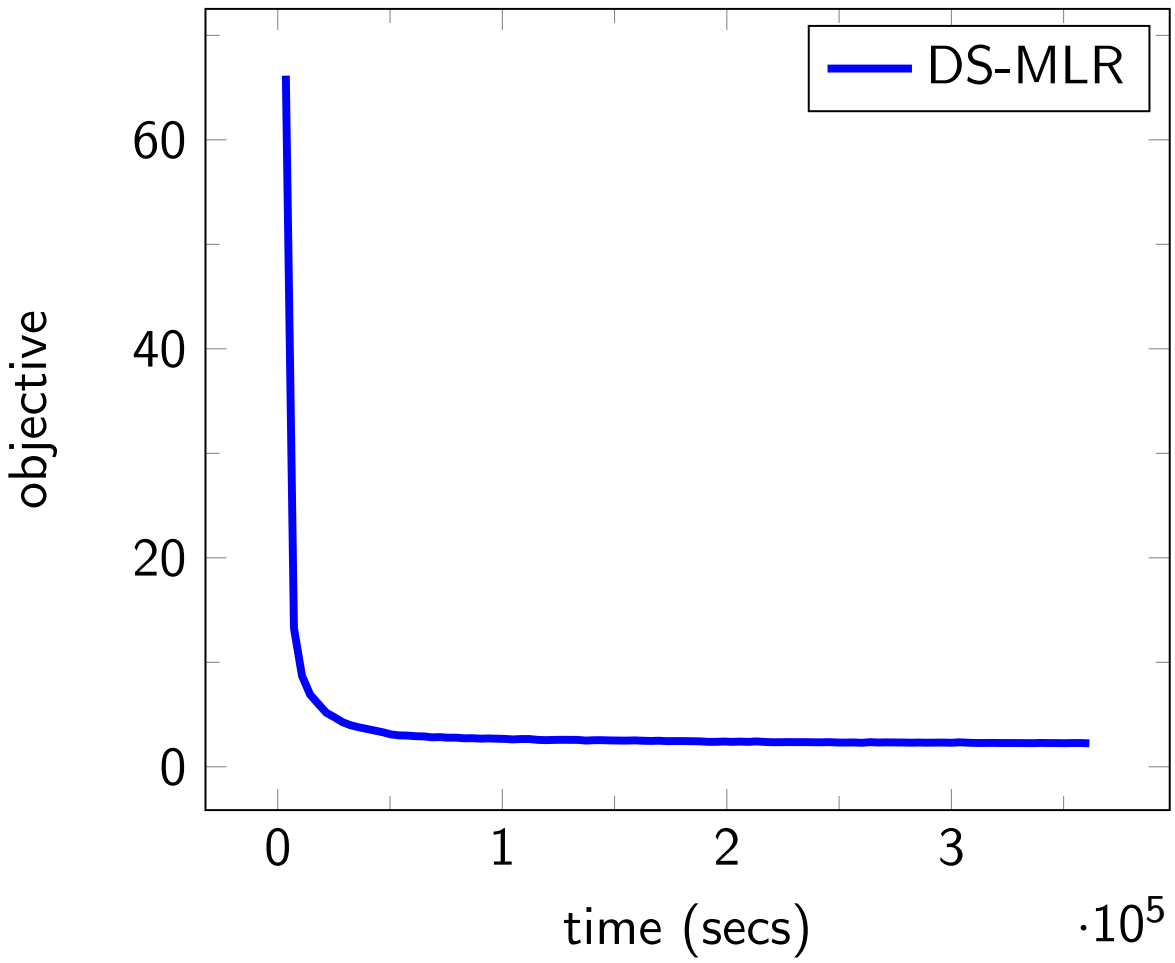


Experiments: Multi Machine



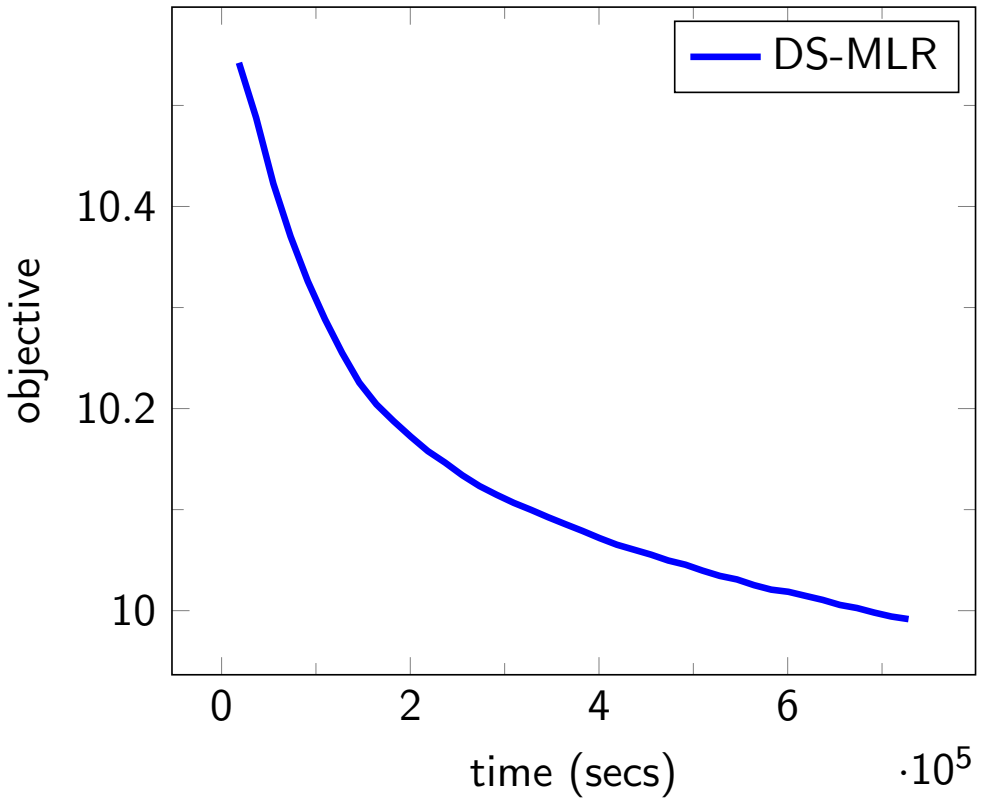
Experiments: Multi Machine Dense Dataset

YouTube-Video, **Data=76 GB**, Model=43 MB,
machines=4, threads=260



Experiments: Multi Machine (Nothing fits in memory!)

Reddit-Full, **Data=228 GB**, **Model=358 GB**,
machines=40, threads=250

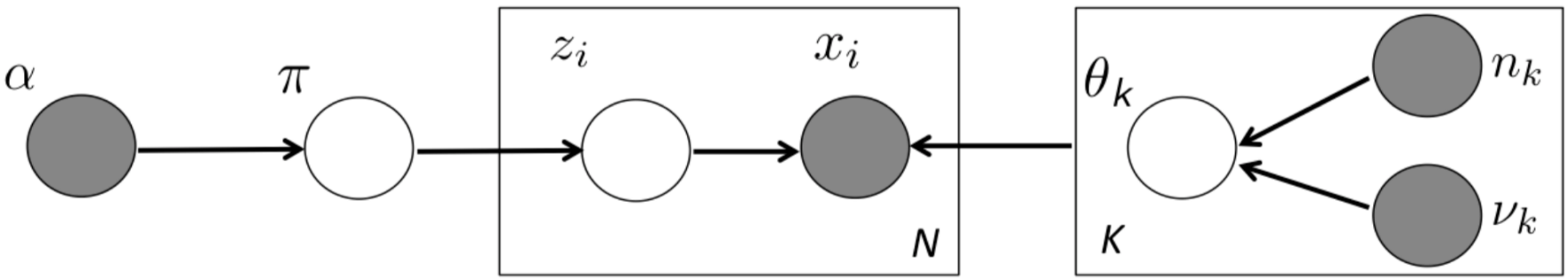


211 million examples, 44 billion parameters (# features $\times$ # classes)

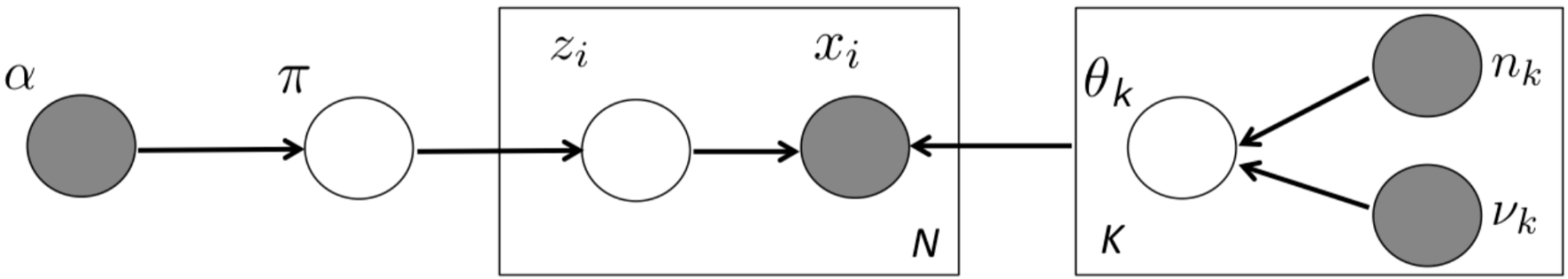
Outline

1. Introduction
2. Distributed Parameter Estimation
3. Thesis Roadmap
   - Multinomial Logistic Regression (KDD 2019)
   - Mixture Models (AISTATS 2019)
   - Latent Collaborative Retrieval (NIPS 2014)
4. Conclusion and Future Directions
5. Appendix

Mixture Of Exponential Families



Mixture Of Exponential Families



- k-means
- Gaussian Mixture Models (GMM)
- Latent Dirichlet Allocation (LDA)
- Stochastic Mixed Membership Models

Generative Model

Generative Model

- **Draw Cluster Proportions π**

$$p(\pi|\alpha) = \text{Dirichlet}(\alpha)$$

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- **Draw Cluster Parameters** for $k = 1, \ldots, K$

$$p(\theta_k|n_k, \nu_k) = \exp\left(\langle n_k \cdot \nu_k, \theta_k \rangle - n_k \cdot g(\theta_k) - h(n_k, \nu_k)\right)$$

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- **Draw Observations** for $i = 1, \ldots, N$

$$p(z_i|\pi) = \text{Multinomial}(\pi)$$

$$p(x_i|z_i, \theta) = \exp\left(\langle \phi(x_i, z_i), \theta_{z_i} \rangle - g(\theta_{z_i})\right)$$

Generative Model

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$$p(\pi|\alpha) = \text{Dirichlet}(\alpha)$$

- **Draw Cluster Parameters** for $k = 1, \ldots, K$

$$p(\theta_k|n_k, \nu_k) = \exp\left(\langle n_k \cdot \nu_k, \theta_k \rangle - n_k \cdot g(\theta_k) - h(n_k, \nu_k)\right)$$

- **Draw Observations** for $i = 1, \ldots, N$

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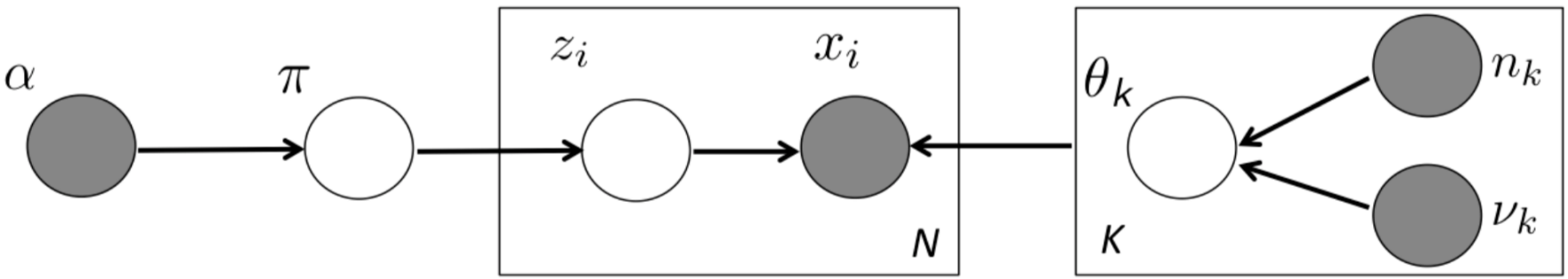
$$p(x_i|z_i, \theta) = \exp\left(\langle \phi(x_i, z_i), \theta_{z_i} \rangle - g(\theta_{z_i})\right)$$

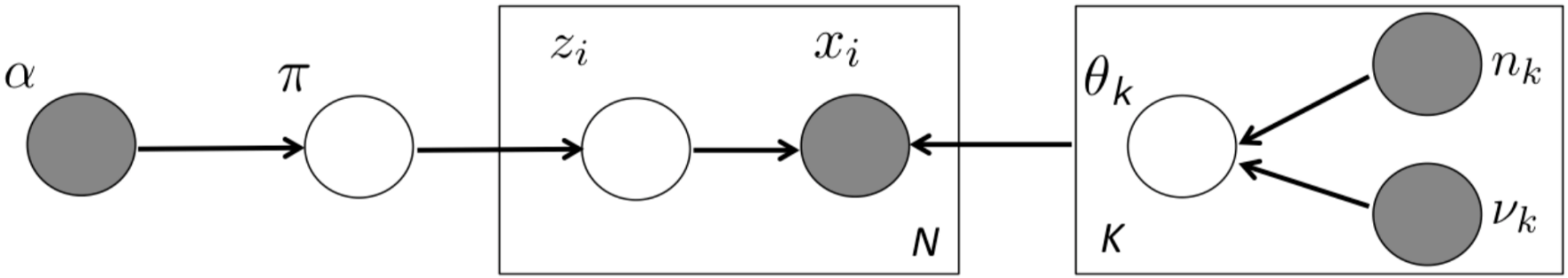
- $p(\theta_k|n_k, \nu_k)$ is conjugate to $p(x_i|z_i = k, \theta_k)$
- $p(\pi|\alpha)$ is conjugate to $p(z_i|\pi)$

Joint Distribution



$$p(\pi, \theta, z, x|\alpha, n, \nu) = p(\pi|\alpha) \cdot \prod_{k=1}^{K} p(\theta_k|n_k, \nu_k) \cdot \prod_{i=1}^{N} p(z_i|\pi) \cdot p(x_i|z_i, \theta)$$

Joint Distribution



$$p(\pi, \theta, z, x|\alpha, n, \nu) = p(\pi|\alpha) \cdot \prod_{k=1}^{K} p(\theta_k|n_k, \nu_k) \cdot \prod_{i=1}^{N} p(z_i|\pi) \cdot p(x_i|z_i, \theta)$$

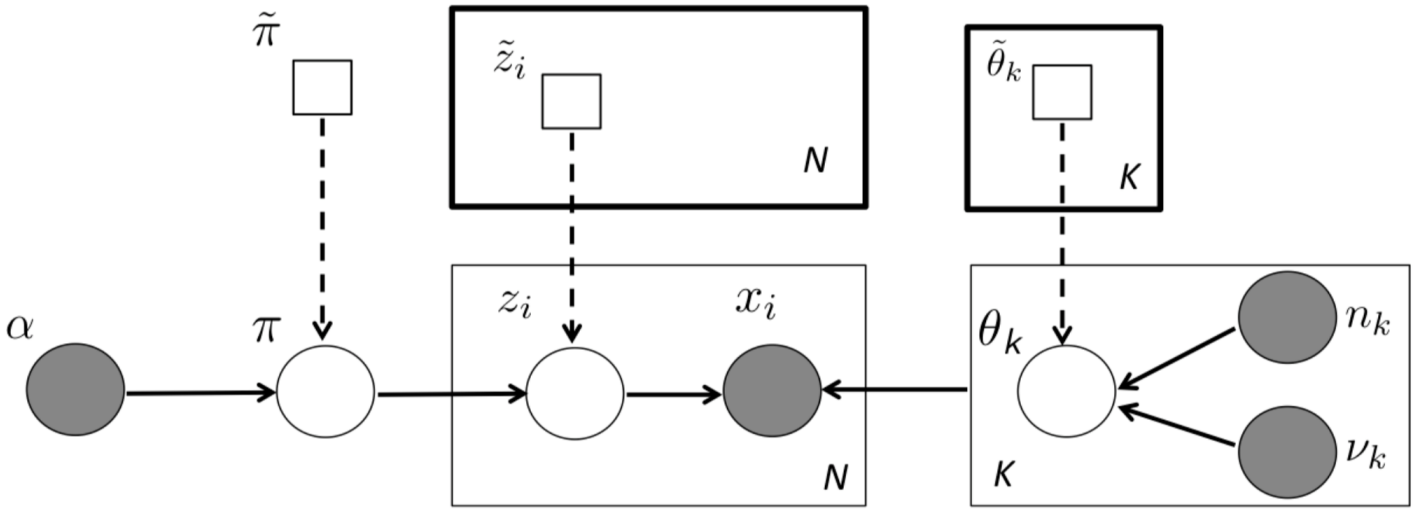
Want to estimate the posterior: $p(\pi, \theta, z|x, \alpha, n, \nu)$

Variational Inference [Blei et al., 2016]

Approximate $p(\pi, \theta, z | x, \alpha, n, \nu)$ with a simpler (factorized) distribution q

$$q\left(\pi, \theta, z | \tilde{\pi}, \tilde{\theta}, \tilde{z}\right) = q(\pi|\tilde{\pi}) \cdot \prod_{k=1}^{K} q\left(\theta_k | \tilde{\theta}_k\right) \cdot \prod_{i=1}^{N} q(z_i | \tilde{z}_i)$$

Variational Inference



Evidence Based Lower Bound (ELBO)

$$\mathcal{L}\left(\tilde{\pi}, \tilde{\theta}, \tilde{z}\right) = \mathbb{E}_{q\left(\pi,\theta,z|\tilde{\pi},\tilde{\theta},\tilde{z}\right)}\left[\log p\left(\pi, \theta, z, x|\alpha, n, \nu\right)\right] - \mathbb{E}_{q\left(\pi,\theta,z|\tilde{\pi},\tilde{\theta},\tilde{z}\right)}\left[\log q\left(\pi, \theta, z|\tilde{\pi}, \tilde{\theta}, \tilde{z}\right)\right]$$

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Variational Inference (VI)

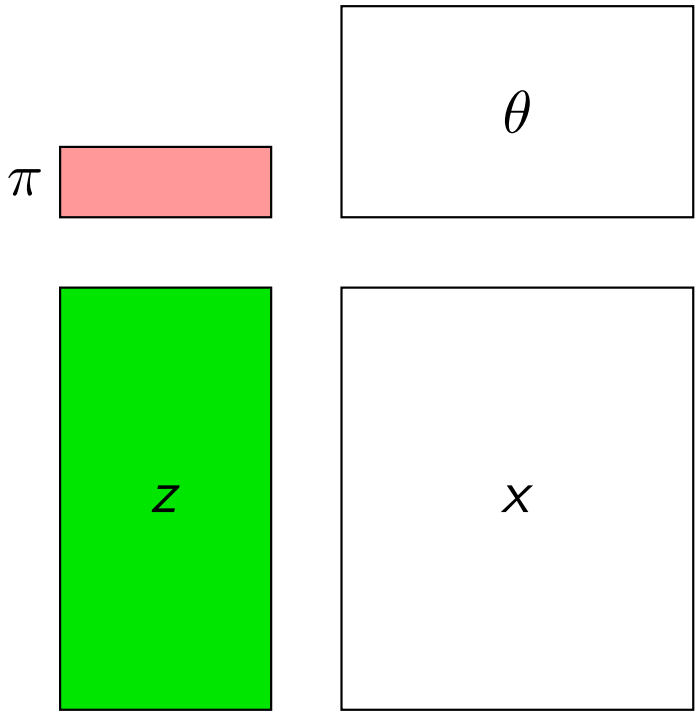
- M-step
  - $\max_{\tilde{\pi}} \mathcal{L}\left(\tilde{\pi}, \tilde{\theta}, \tilde{z}\right)$
  - $\max_{\tilde{\theta}} \mathcal{L}\left(\tilde{\pi}, \tilde{\theta}, \tilde{z}\right)$
- E-step
  - $\max_{\tilde{z}} \mathcal{L}\left(\tilde{\pi}, \tilde{\theta}, \tilde{z}\right)$

Variational Inference: M-step

- $\max_{\tilde{\pi}} \mathcal{L}\left(\tilde{\pi}, \tilde{\theta}, \tilde{z}\right)$

$$\tilde{\pi}_k = \alpha + \sum_{i=1}^{N} \tilde{z}_{i,k}$$

Access Pattern of Variables: $\tilde{\pi}$ update



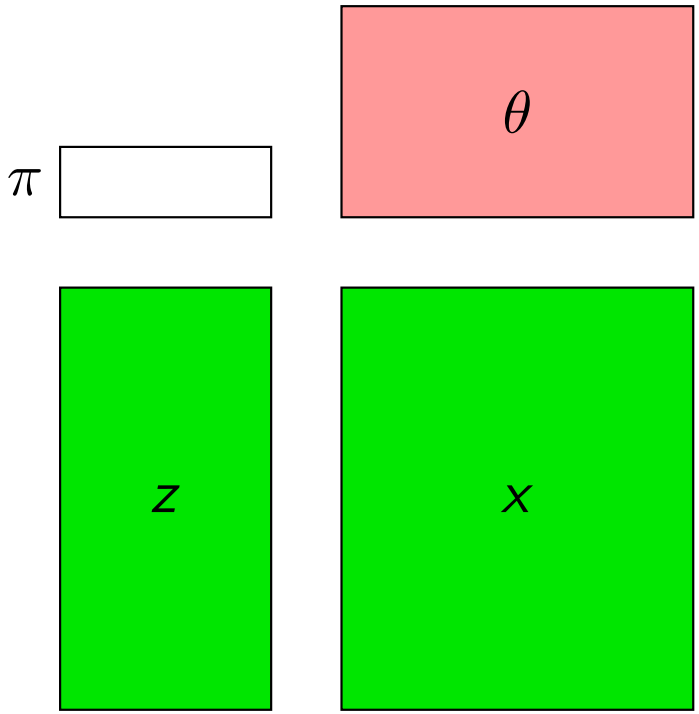
Variational Inference: M-step

- $\max_{\tilde{\theta}} \mathcal{L}\left(\tilde{\pi}, \tilde{\theta}, \tilde{z}\right)$

$$\tilde{n}_k = n_k + \sum_{i=1}^{N} \tilde{z}_{i,k}$$

$$\tilde{\nu}_k = n_k \cdot \nu_k + \sum_{i=1}^{N} \tilde{z}_{i,k} \cdot \phi\left(x_i, k\right)$$

Access Pattern of Variables: $\tilde{\theta}$ update

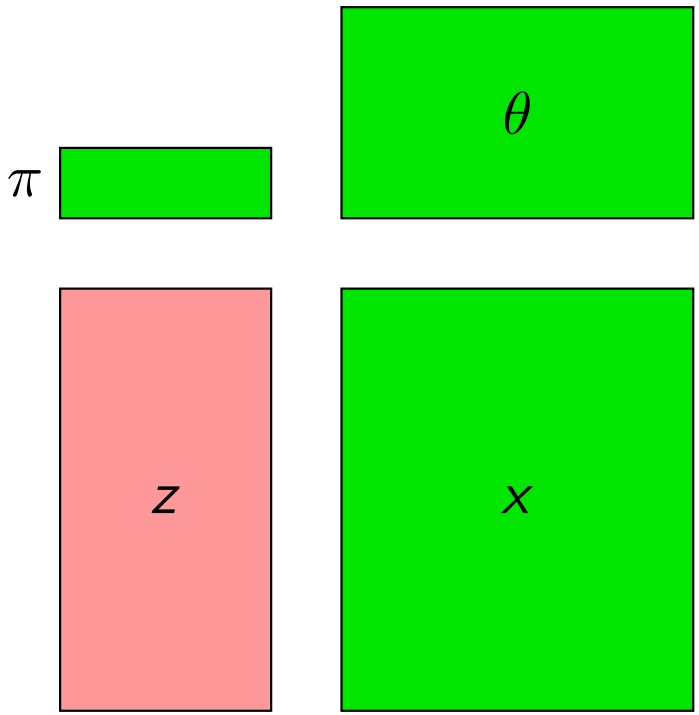


Variational Inference: E-step

- $\max_{\tilde{z}} \mathcal{L}\left(\tilde{\pi}, \tilde{\theta}, \tilde{z}\right)$

$$\tilde{z}_{i,k} \propto \exp\left(\psi\left(\tilde{\pi}_k\right) - \psi\left(\sum_{k'=1}^{K} \tilde{\pi}_{k'}\right) + \left\langle \phi\left(x_i, k\right), \mathbb{E}_{q\left(\theta_k|\tilde{\theta}_k\right)}\left[\theta_k\right]\right\rangle - \mathbb{E}_{q\left(\theta_k|\tilde{\theta}_k\right)}\left[g\left(\theta_k\right)\right]\right)$$

Access Pattern of Variables: $\tilde{z}$ update



Stochastic Variational Inference (SVI) [Hoffman et al., 2013]

- M-step
  - $\max_{\tilde{\pi}} \mathcal{L}\left(\tilde{\pi}, \tilde{\theta}, \tilde{z}\right)$
  - $\max_{\tilde{\theta}} \mathcal{L}\left(\tilde{\pi}, \tilde{\theta}, \tilde{z}\right)$
- E-step
  - $\max_{\tilde{z}_i} \mathcal{L}\left(\tilde{\pi}, \tilde{\theta}, \tilde{z}_{\setminus i}, \tilde{z}_i\right)$

Stochastic Variational Inference (SVI) [Hoffman et al., 2013]

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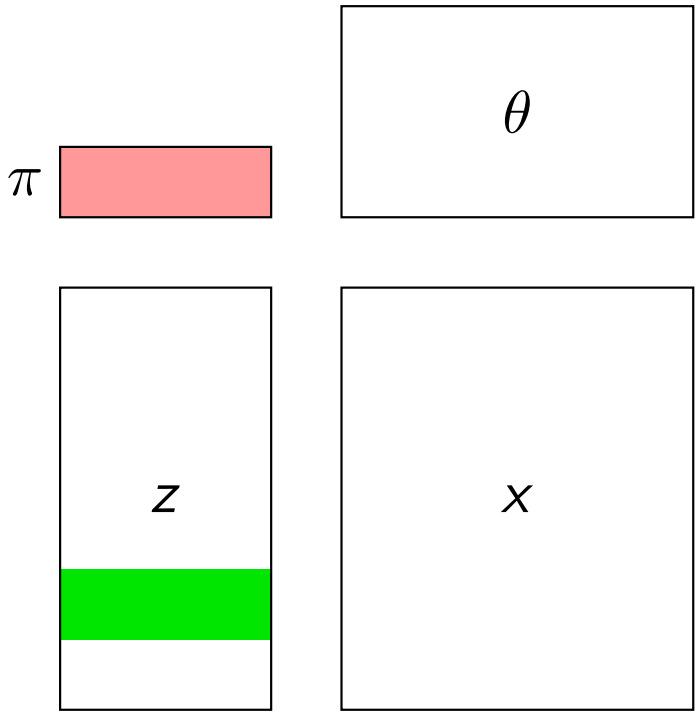
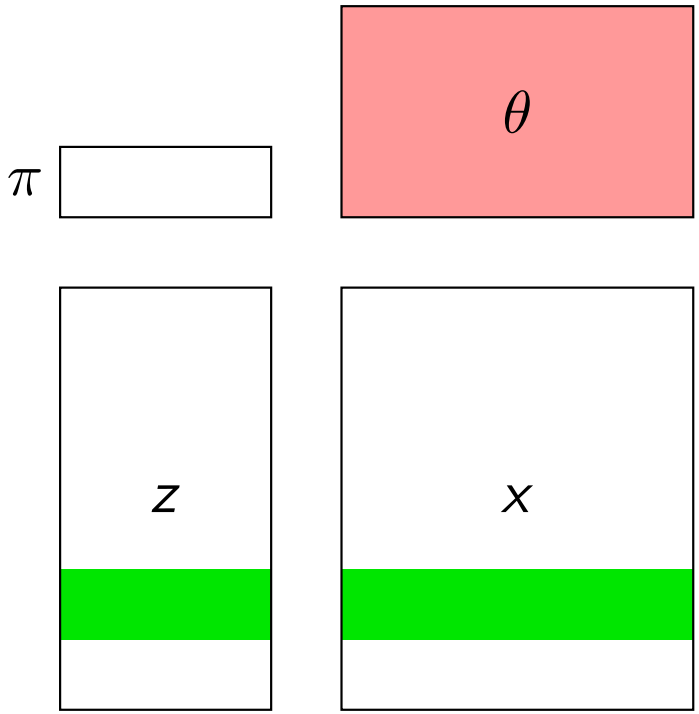
  - $\max_{\tilde{\theta}} \mathcal{L}\left(\tilde{\pi}, \tilde{\theta}, \tilde{z}\right)$
- E-step
  - $\max_{\tilde{z}_i} \mathcal{L}\left(\tilde{\pi}, \tilde{\theta}, \tilde{z}_{\setminus i}, \tilde{z}_i\right)$

More frequent updates to global variables $\Longrightarrow$ Faster convergence

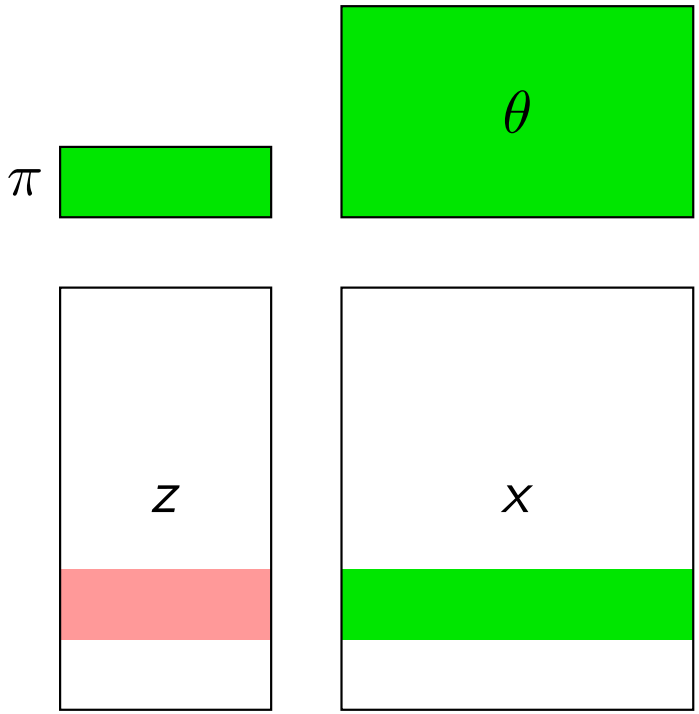
Access Pattern of Variables: $\tilde{\pi}$ update



Access Pattern of Variables: $\tilde{\theta}$ update



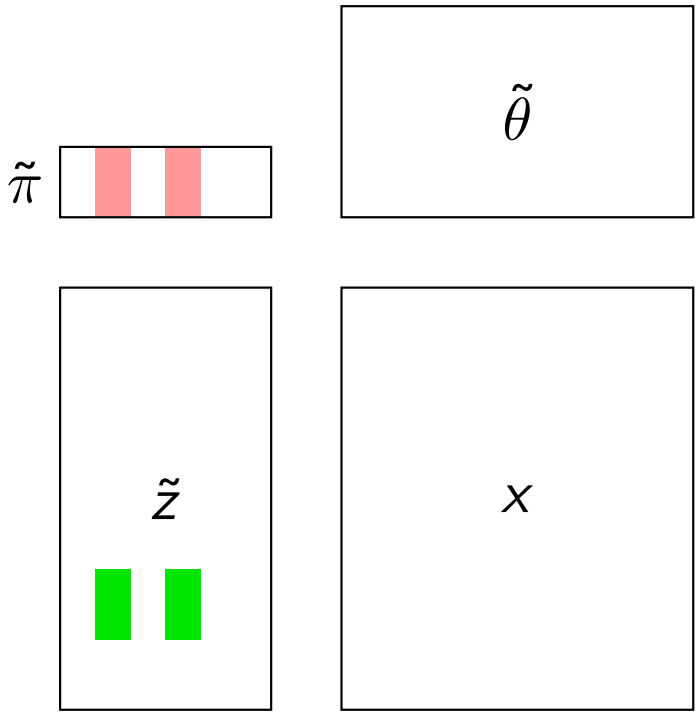
Access Pattern of Variables: $\tilde{z}$ update



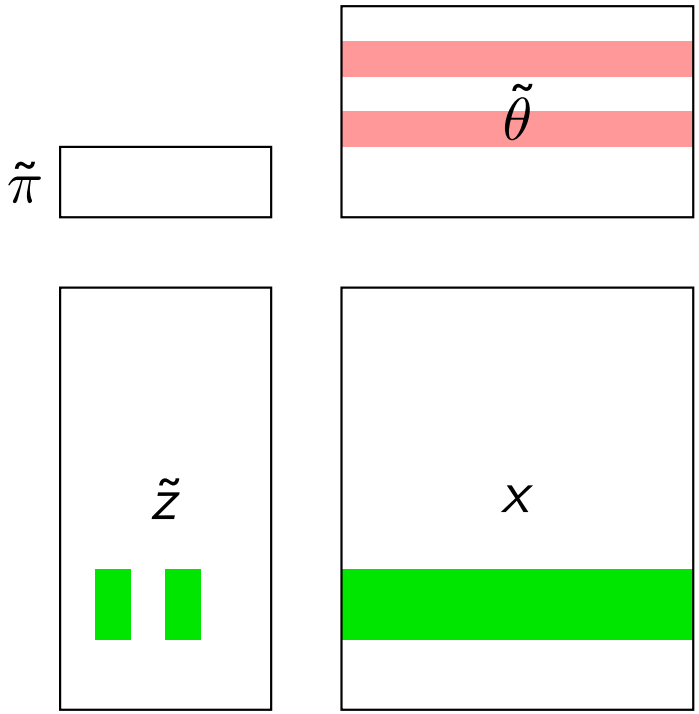
Extreme Stochastic Variational Inference (ESVI)

- M-step
  - $\max_{\tilde{\pi}_k, \tilde{\pi}_{k'}} \mathcal{L}\left([\tilde{\pi}_1, \ldots, \tilde{\pi}_k, \tilde{\pi}_{k+1}, \ldots, \tilde{\pi}_{k'}, \ldots, \tilde{\pi}_K], \tilde{\theta}, \tilde{z}\right)$
  - $\max_{\tilde{\theta}_k, \tilde{\theta}_{k'}} \mathcal{L}\left(\tilde{\pi}, \left[\tilde{\theta}_1, \ldots, \tilde{\theta}_k, \tilde{\theta}_{k+1}, \ldots, \tilde{\theta}_{k'}, \ldots, \tilde{\theta}_K\right], \tilde{z}\right)$
- E-step
  - $\max_{\tilde{z}_{i,k}, \tilde{z}_{i,k'}} \mathcal{L}\left(\tilde{\pi}, \tilde{\theta}, \tilde{z}_{\setminus i}, [\tilde{z}_{i,1}, \ldots, \tilde{z}_{i,k}, \tilde{z}_{i,k+1}, \ldots, \tilde{z}_{i,k'}, \ldots, \tilde{z}_{i,K}]\right)$

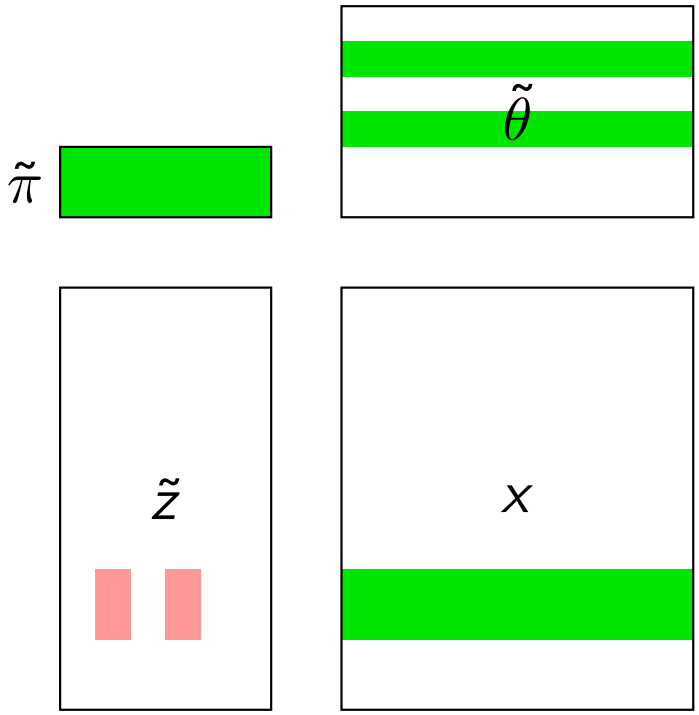
Access Pattern of Variables: $\tilde{\pi}$ update



Access Pattern of Variables: $\tilde{\theta}$ update



Access Pattern of Variables: $\tilde{z}$ update



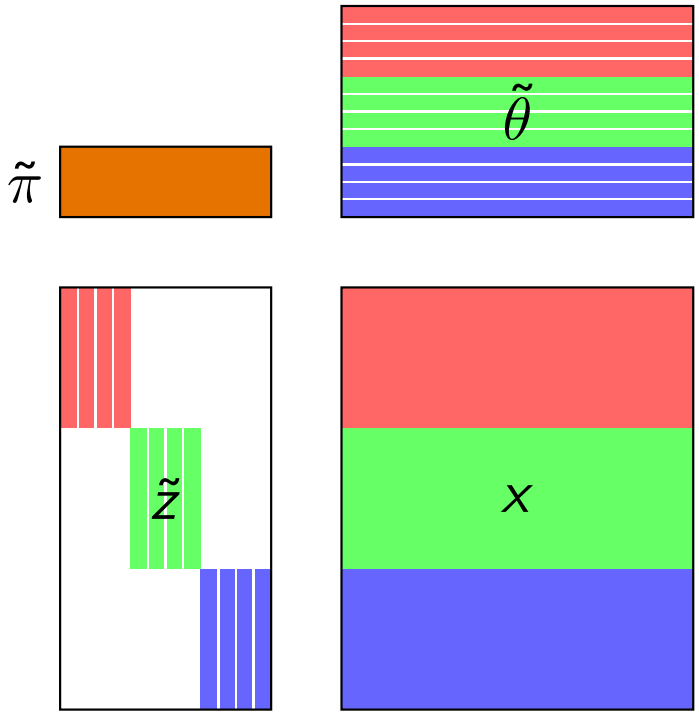
Key Observation

- M-step
  - $\max_{\tilde{\pi}_k, \tilde{\pi}_{k'}} \mathcal{L}\left([\tilde{\pi}_1, \ldots, \tilde{\pi}_k, \tilde{\pi}_{k+1}, \ldots, \tilde{\pi}_{k'}, \ldots, \tilde{\pi}_K], \tilde{\theta}, \tilde{z}\right)$
  - $\max_{\tilde{\theta}_k, \tilde{\theta}_{k'}} \mathcal{L}\left(\tilde{\pi}, \left[\tilde{\theta}_1, \ldots, \tilde{\theta}_k, \tilde{\theta}_{k+1}, \ldots, \tilde{\theta}_{k'}, \ldots, \tilde{\theta}_K\right], \tilde{z}\right)$
- E-step
  - $\max_{\tilde{z}_{i,k}, \tilde{z}_{i,k'}} \mathcal{L}\left(\tilde{\pi}, \tilde{\theta}, \tilde{z}_{\setminus i}, [\tilde{z}_{i,1}, \ldots, \tilde{z}_{i,k}, \tilde{z}_{i,k+1}, \ldots, \tilde{z}_{i,k'}, \ldots, \tilde{z}_{i,K}]\right)$

All the above updates can be done in parallel!

Distributing Computation

NOMAD [Yun et al., 2014]



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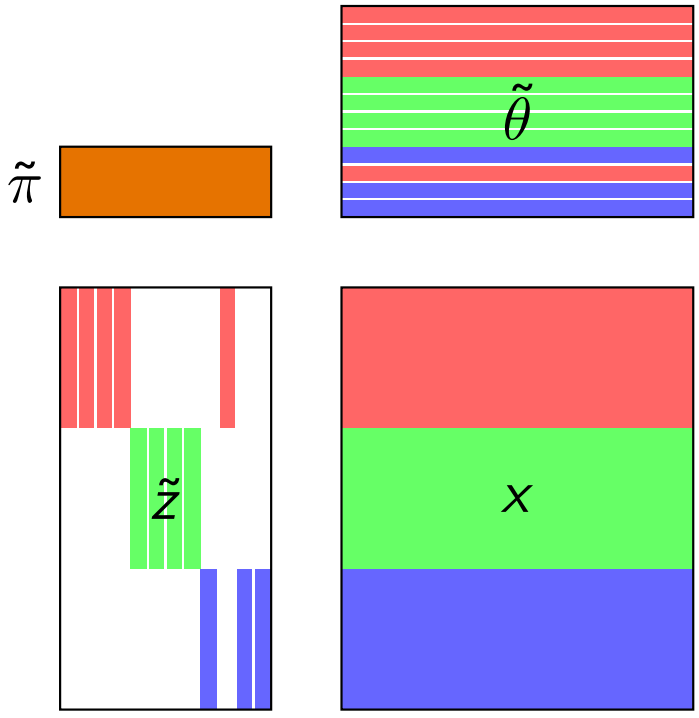

Distributing Computation

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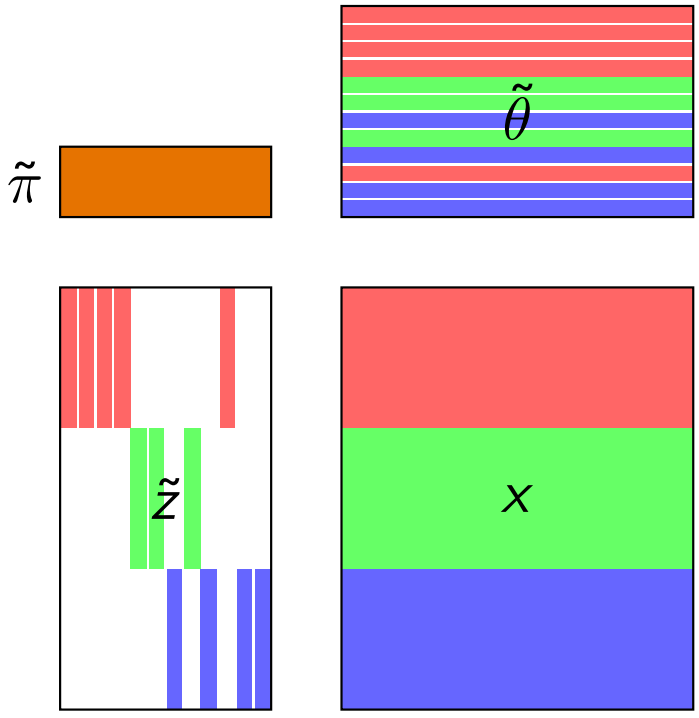

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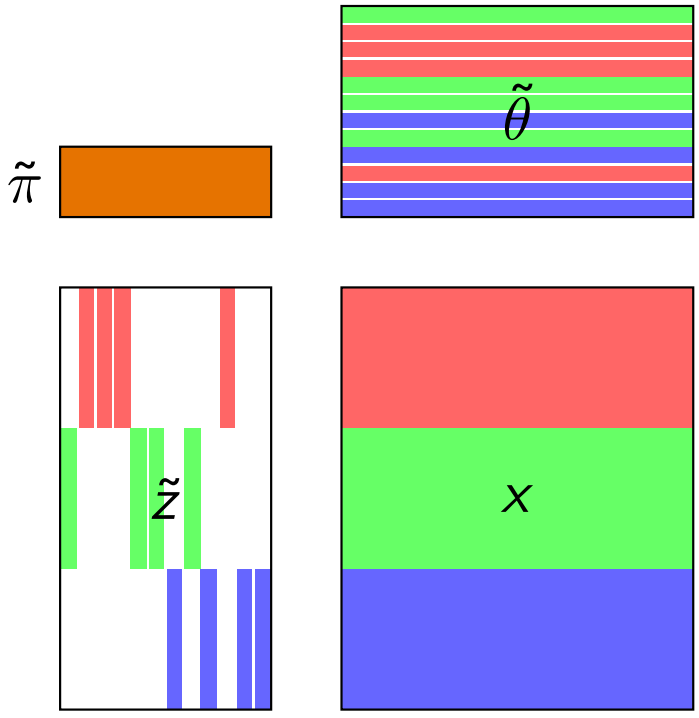
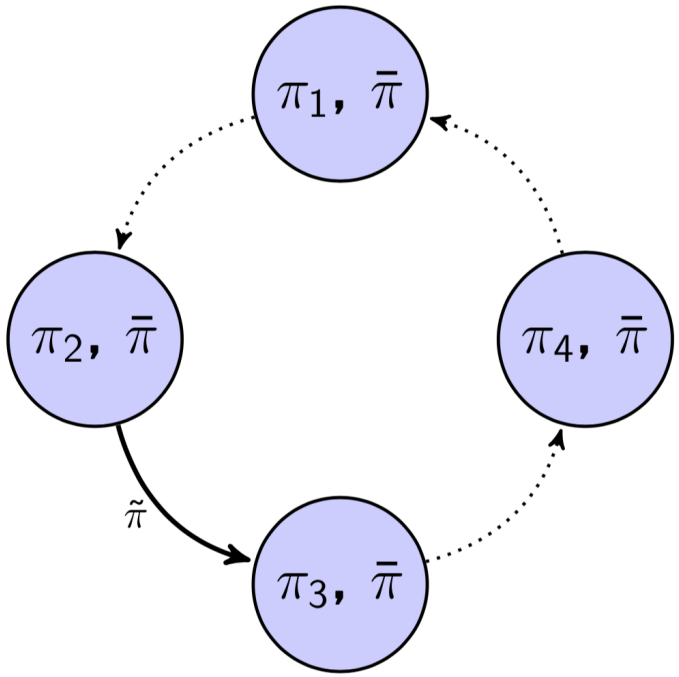


Keeping $\tilde{\pi}$ in Sync

- Single global $\tilde{\pi}$
  - travels among machines
  - broadcasts local delta updates
- Every machine p: $(\pi_p, \bar{\pi})$
  - π_p: local working copy
  - $\bar{\pi}$: snapshot of global $\tilde{\pi}$



$\tilde{\pi} \leftarrow \tilde{\pi} + (\pi_3 - \bar{\pi})$
$\bar{\pi} \leftarrow \tilde{\pi}$
$\pi_3 \leftarrow \tilde{\pi}$

Experiments: Datasets

	# documents	# vocabulary	#words
NIPS	1,312	12,149	1,658,309
Enron	37,861	28,102	6,238,796
Ny Times	298,000	102,660	98,793,316
PubMed	8,200,000	141,043	737,869,083
UMBC-3B	40,599,164	3,431,260	3,013,004,127

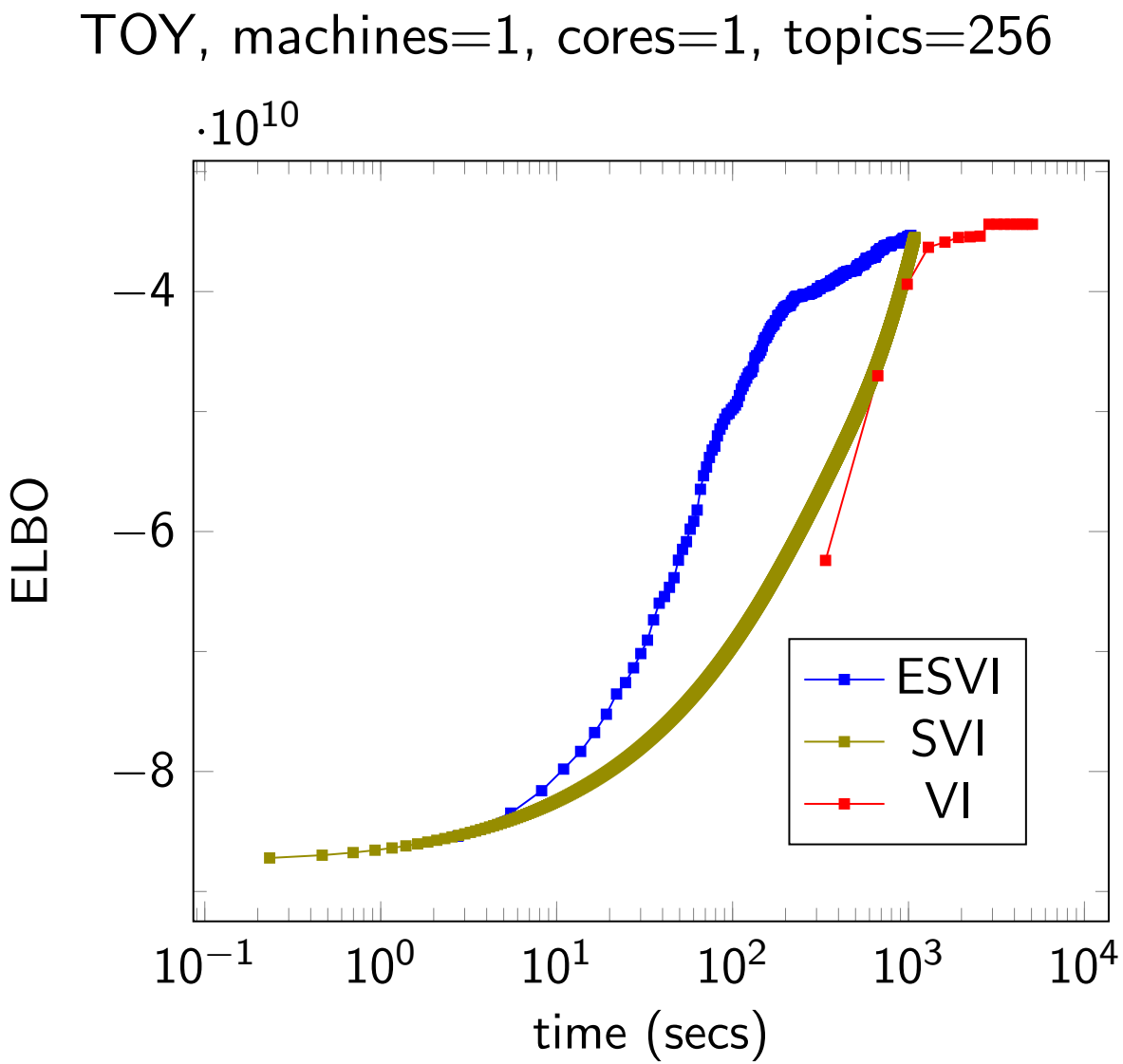
Mixture Models:

- Gaussian Mixture Model (GMM)
- Latent Dirichlet Allocation (LDA)

Methods:

- Variational Inference (VI)
- Stochastic Variational Inference (SVI)
- Extreme Stochastic Variational Inference (ESVI)

Experiments: Single Machine (GMM)



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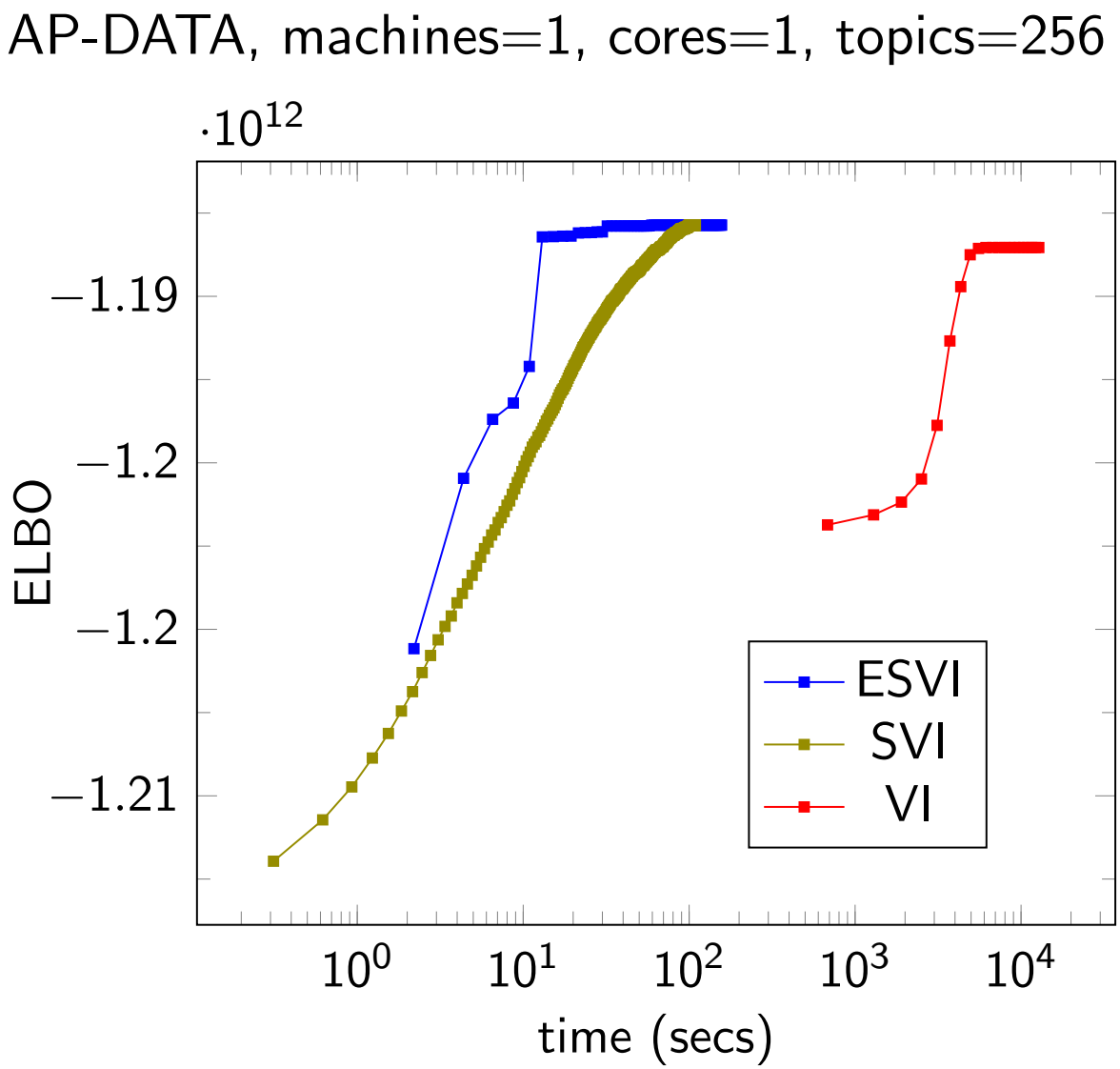
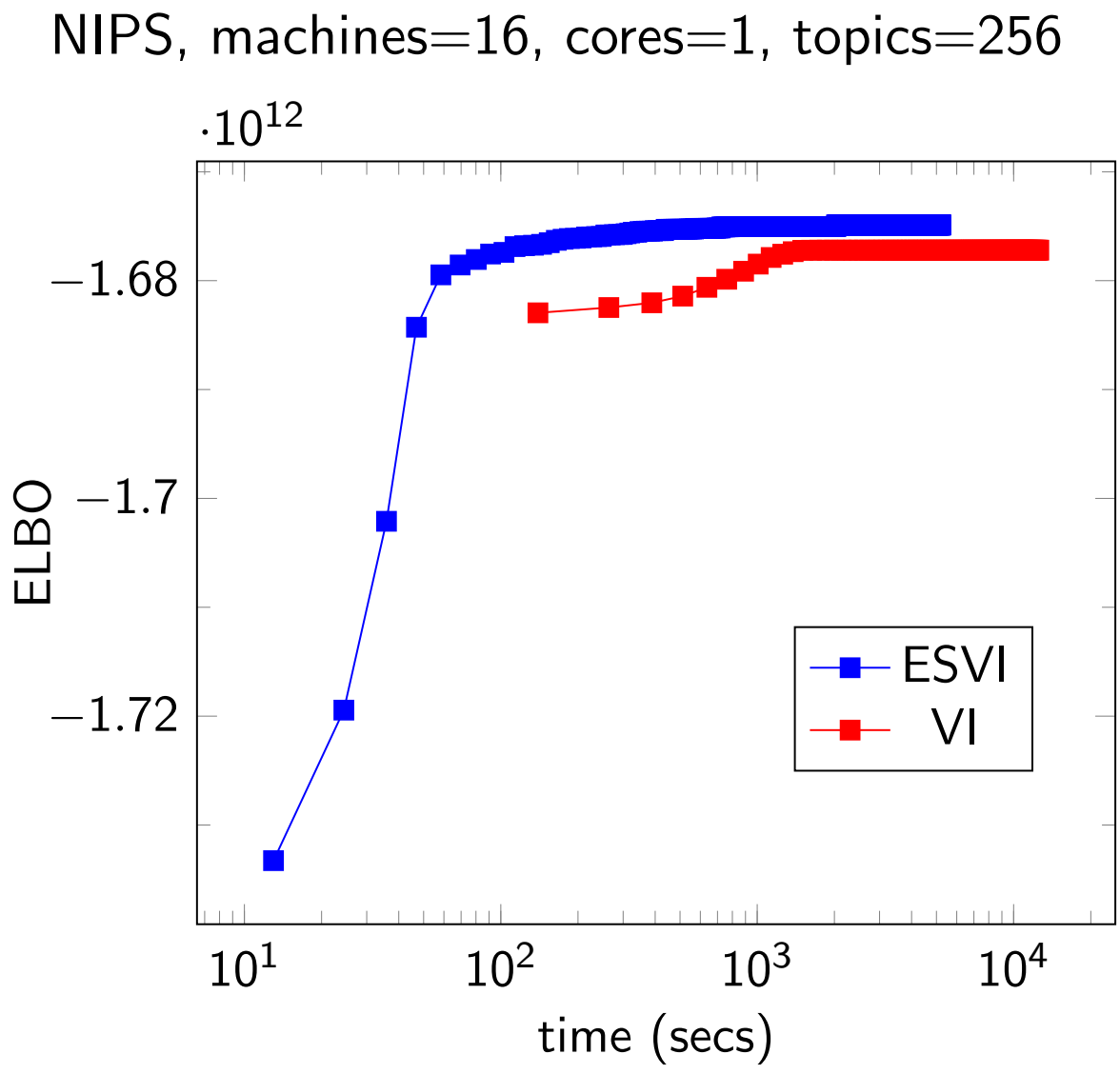
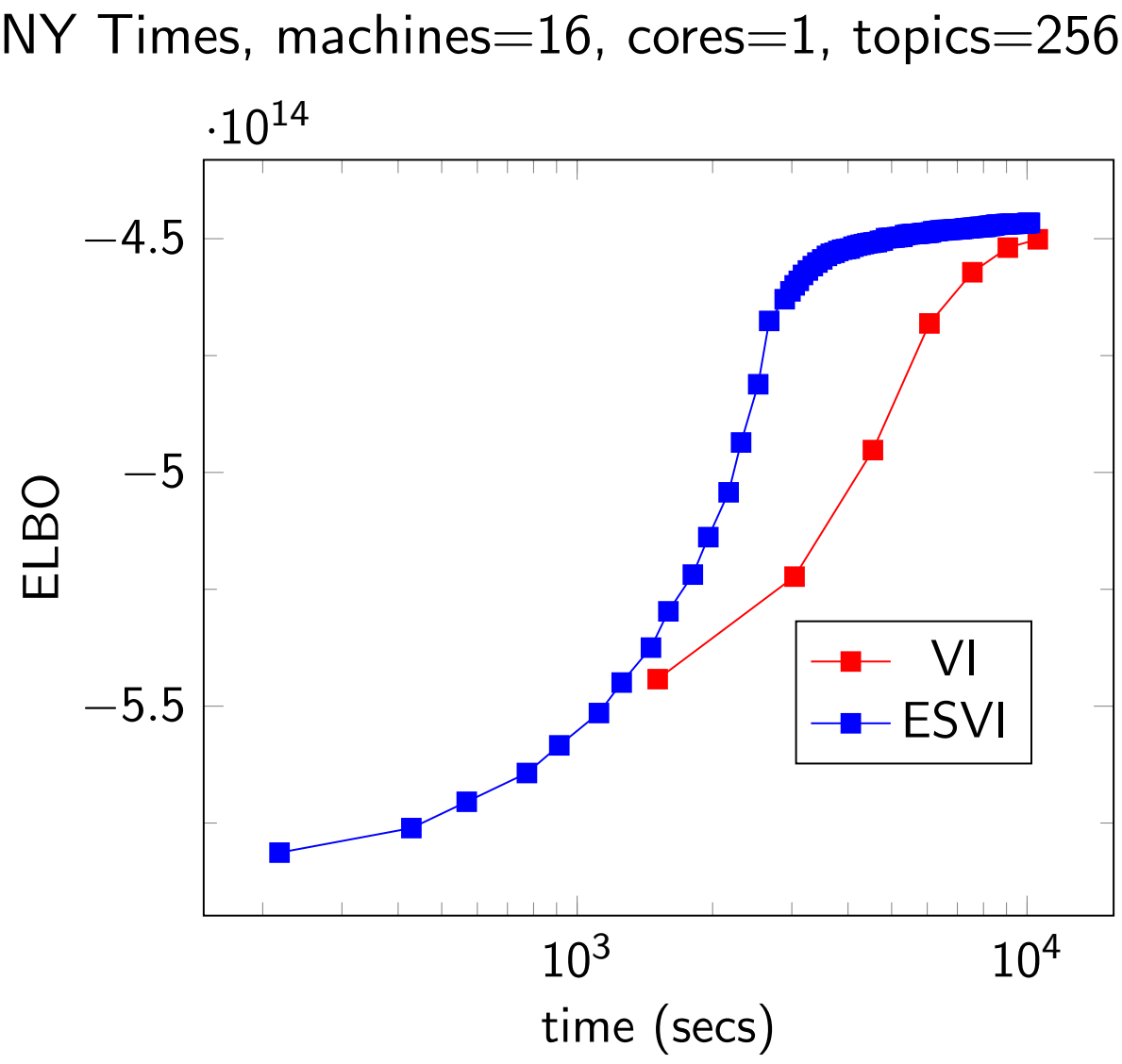


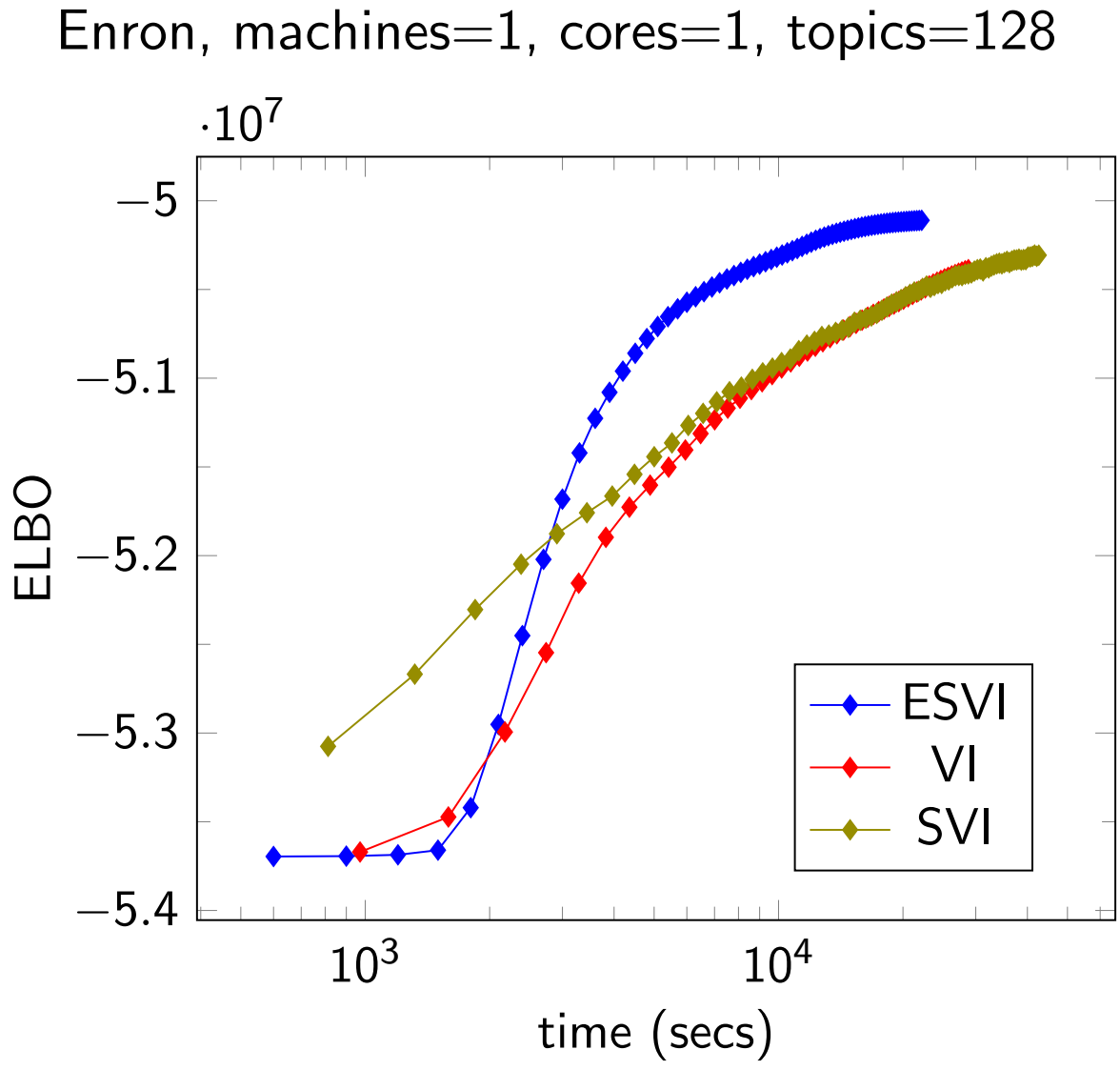
Experiments: Multi Machine (GMM)



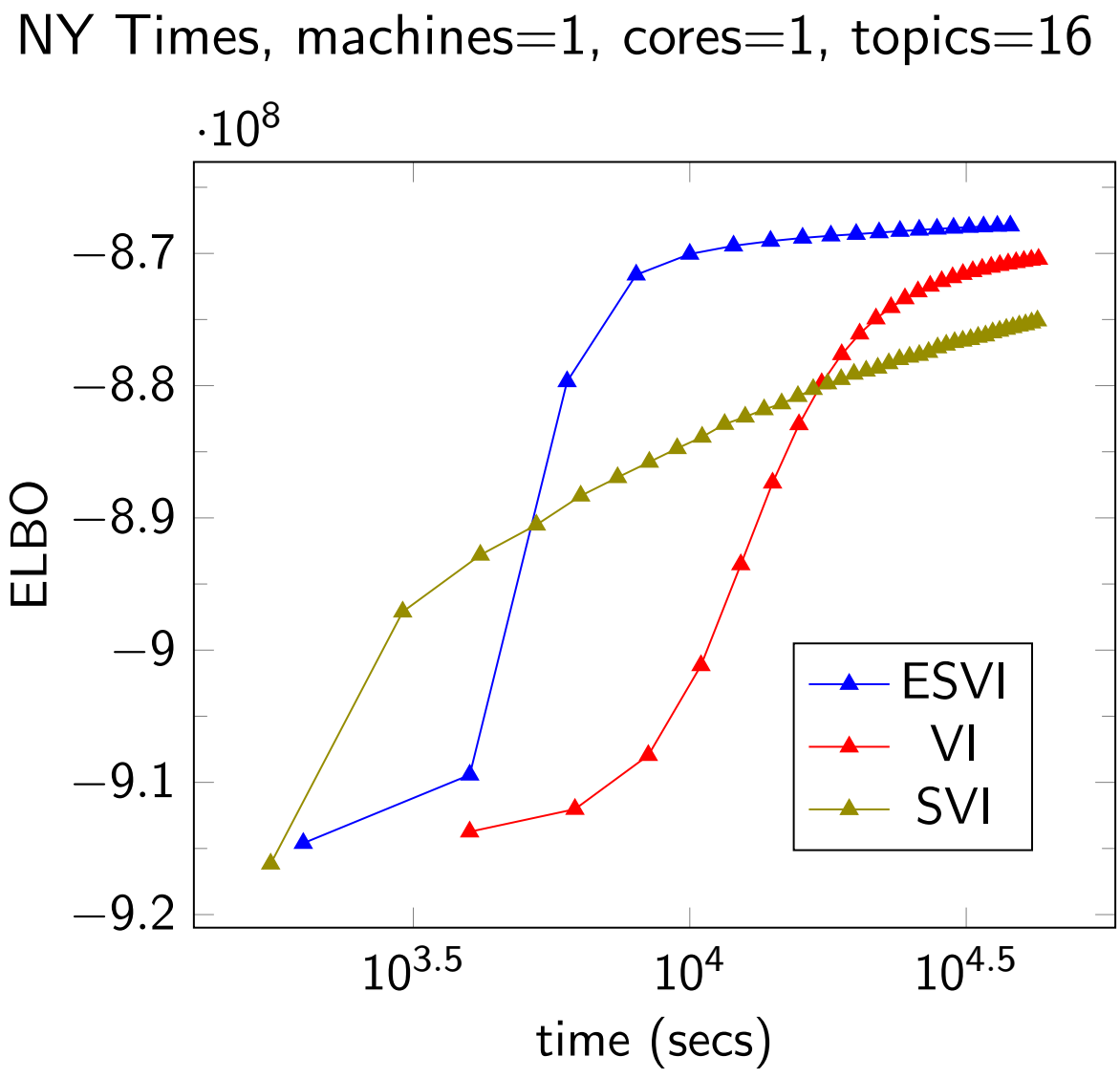
Experiments: Multi Machine (GMM)



Experiments: Single Machine Single Core (LDA)



Experiments: Single Machine Single Core (LDA)



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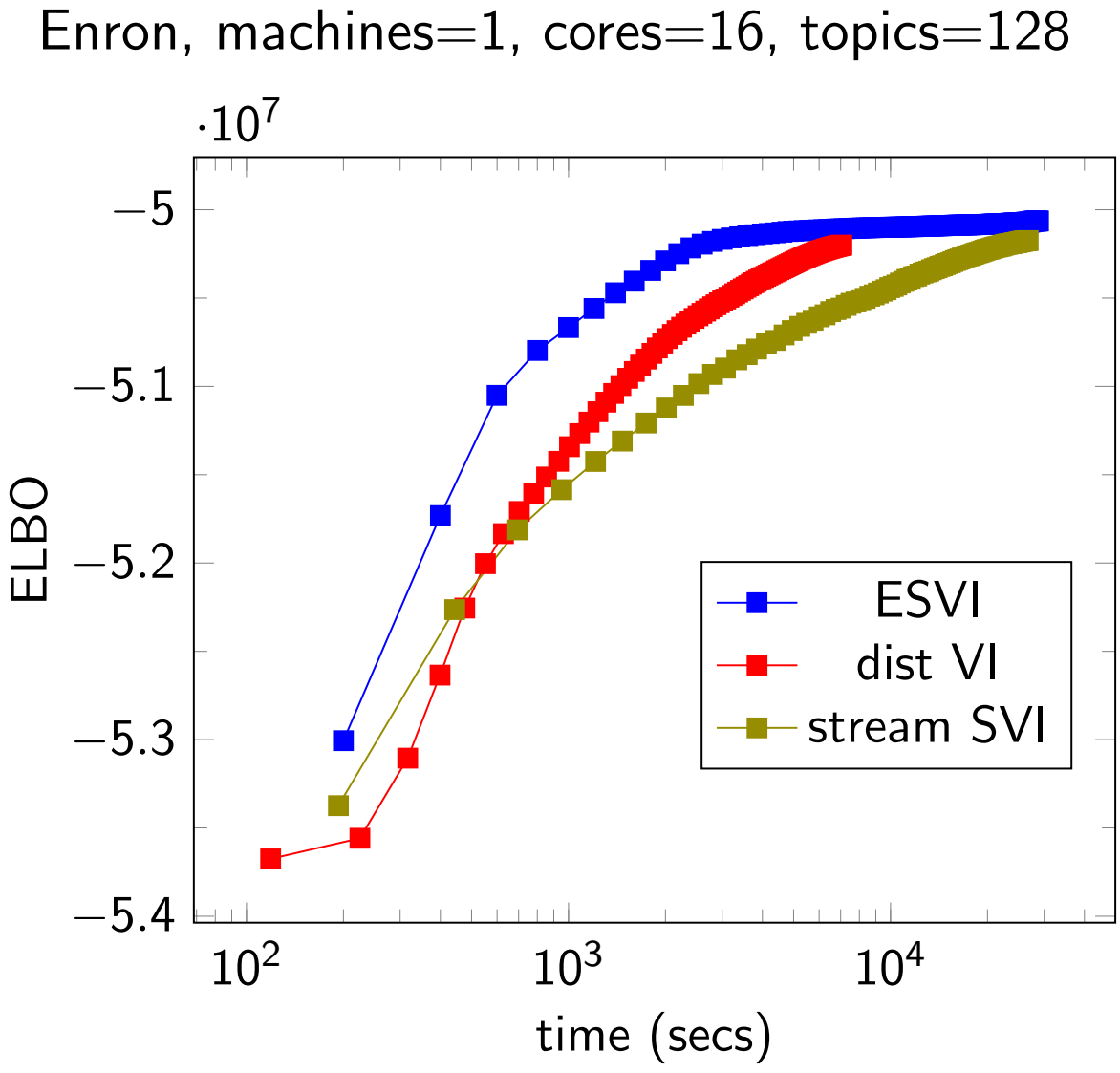

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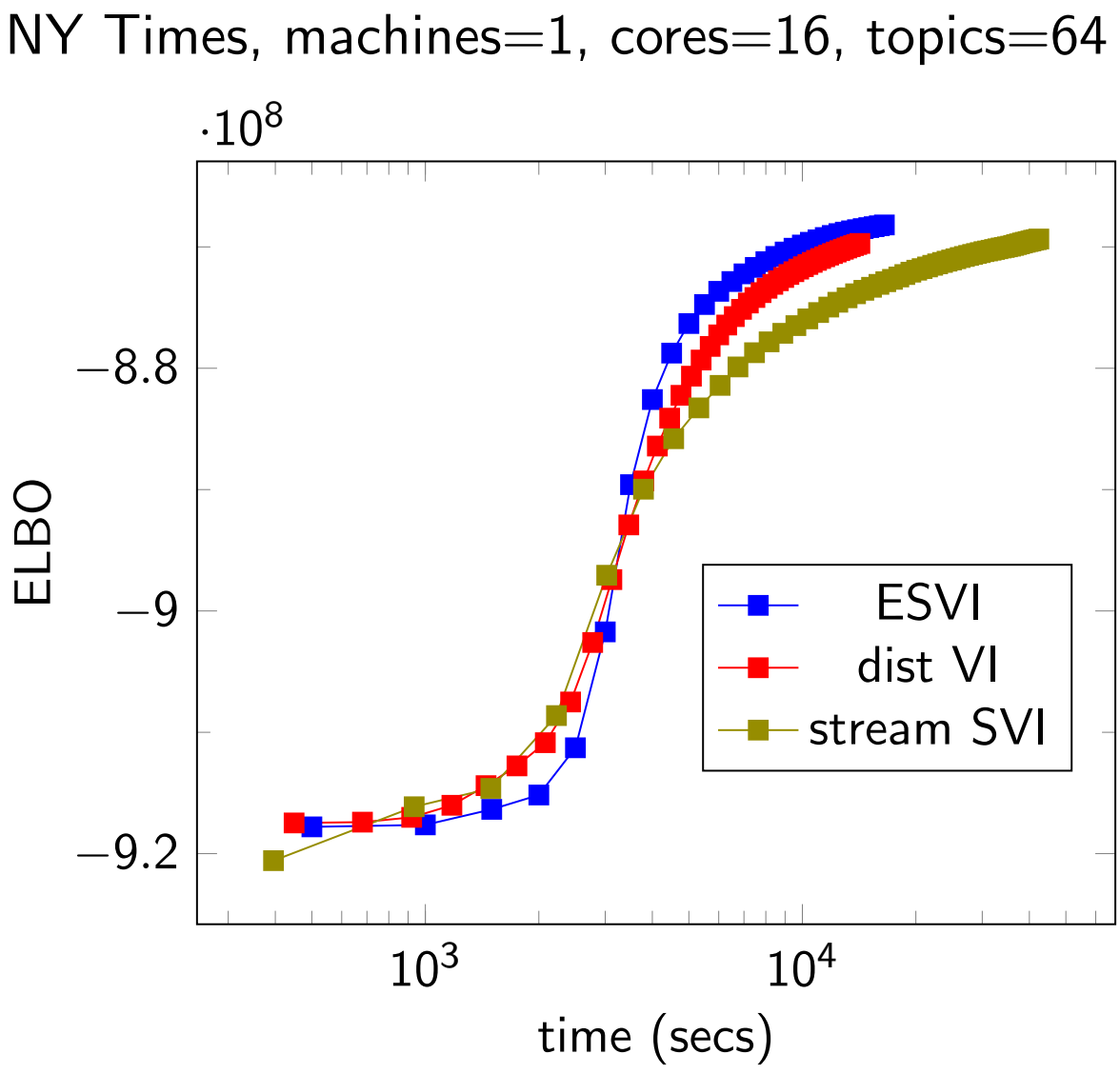

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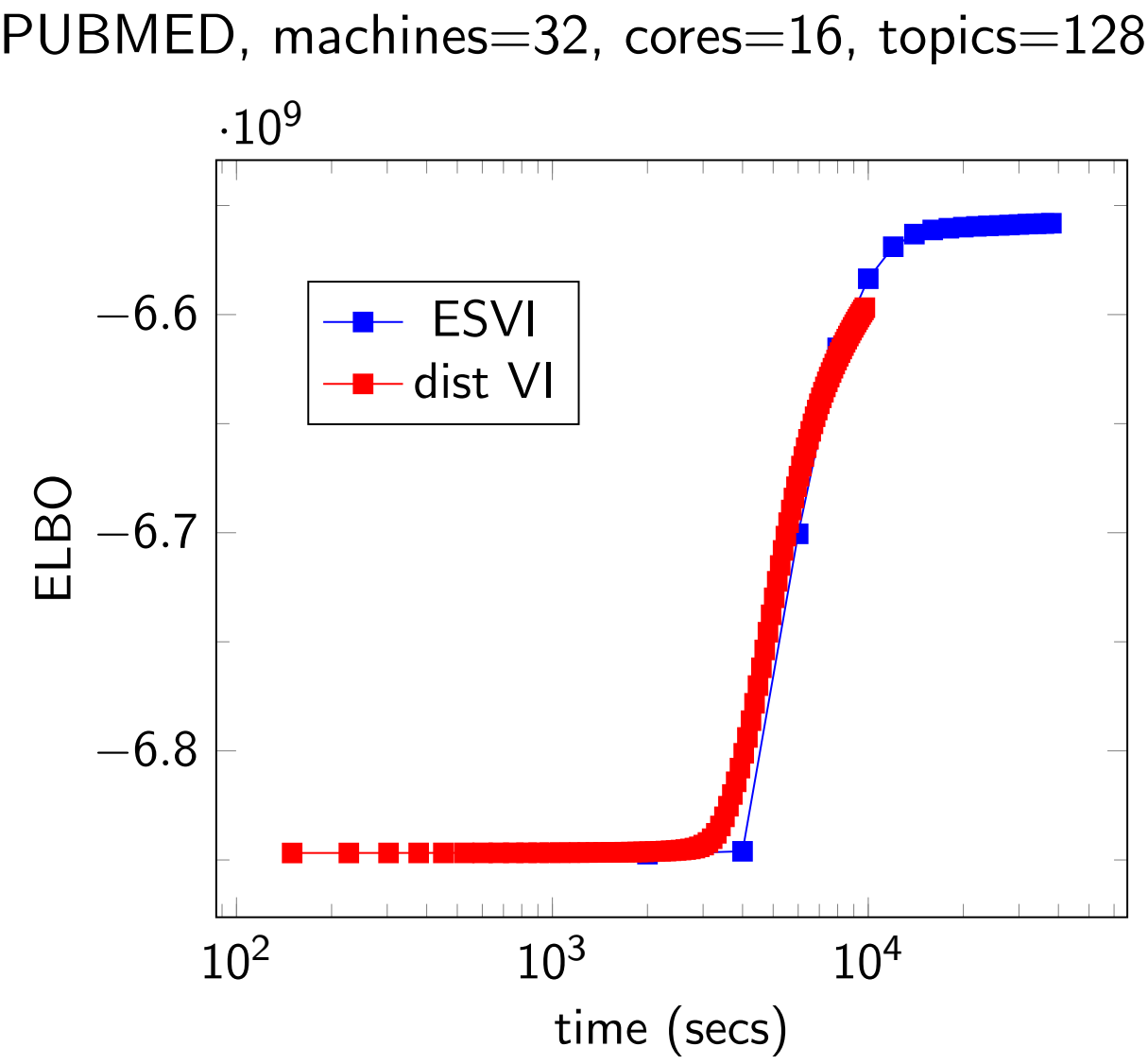

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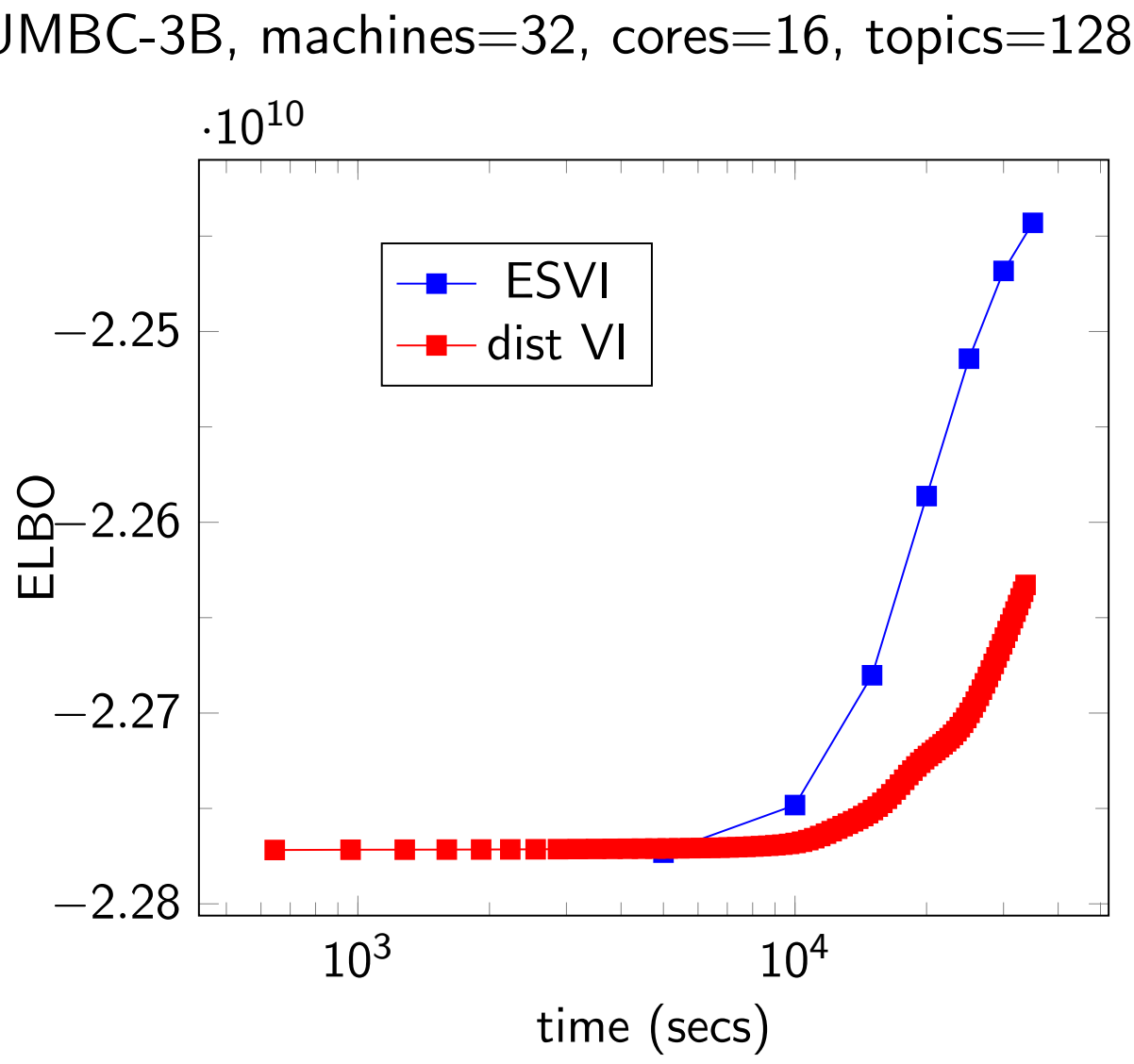
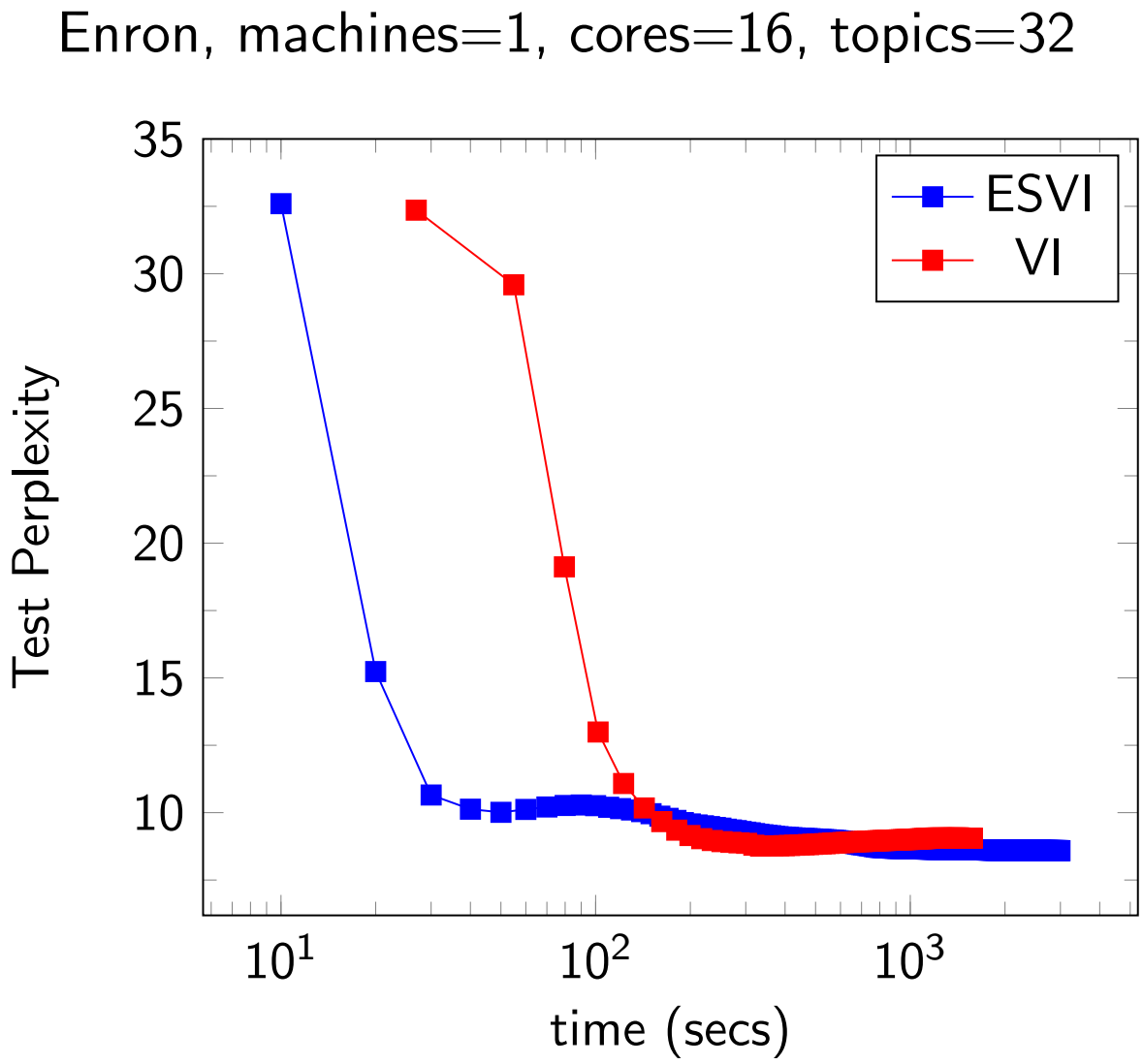


Experiments: Predictive Performance (LDA)



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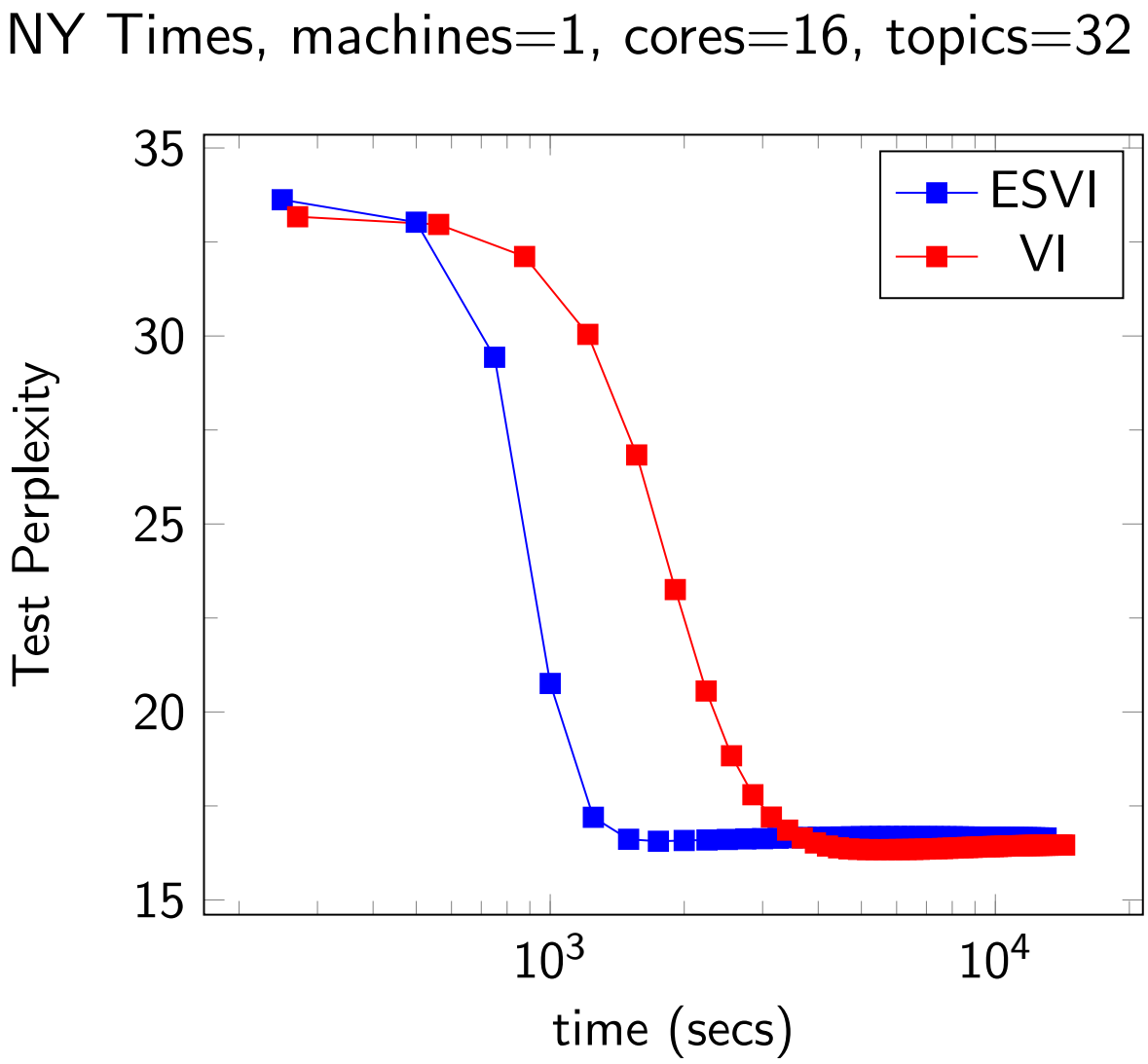

Outline

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2. Distributed Parameter Estimation
3. Thesis Roadmap
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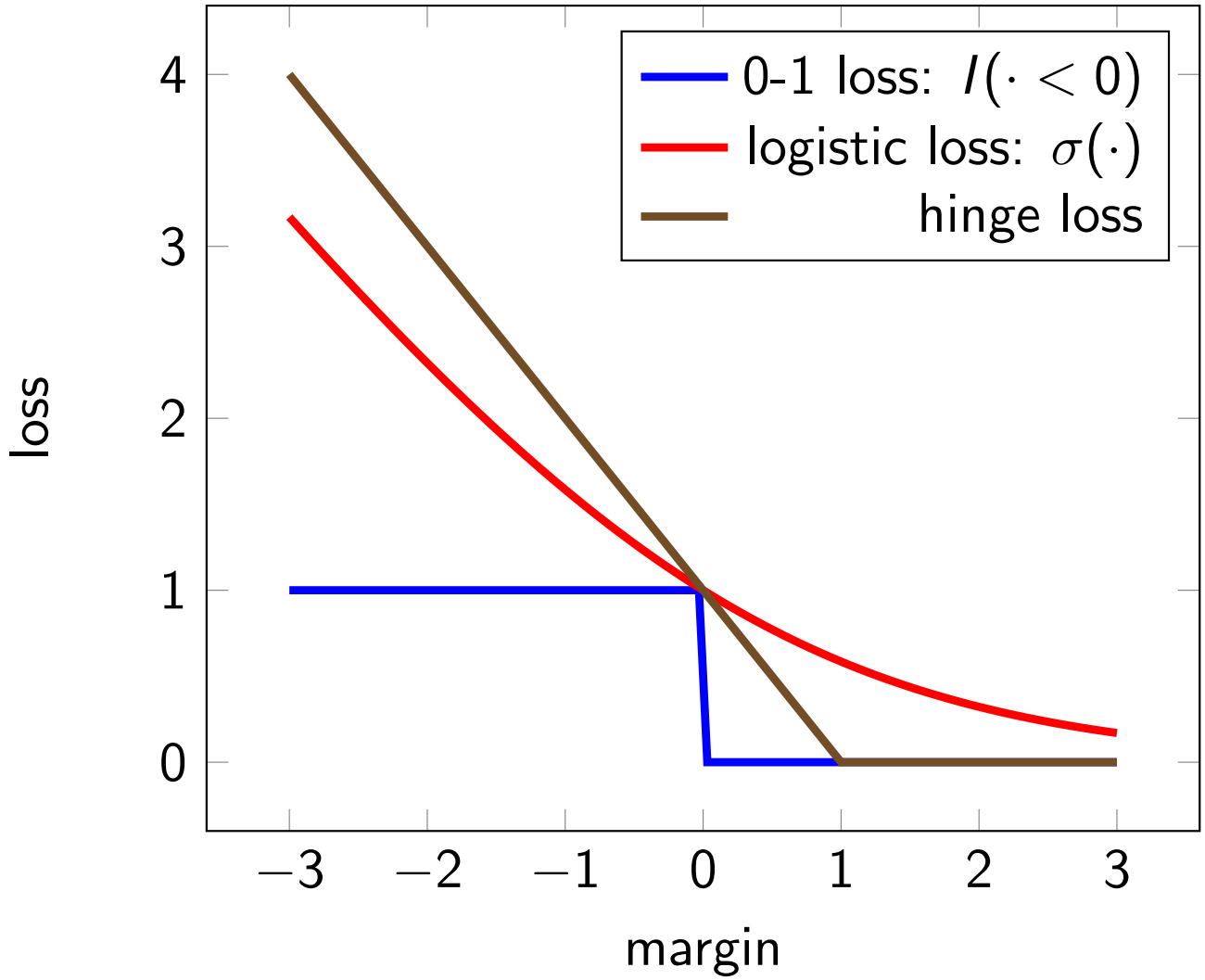
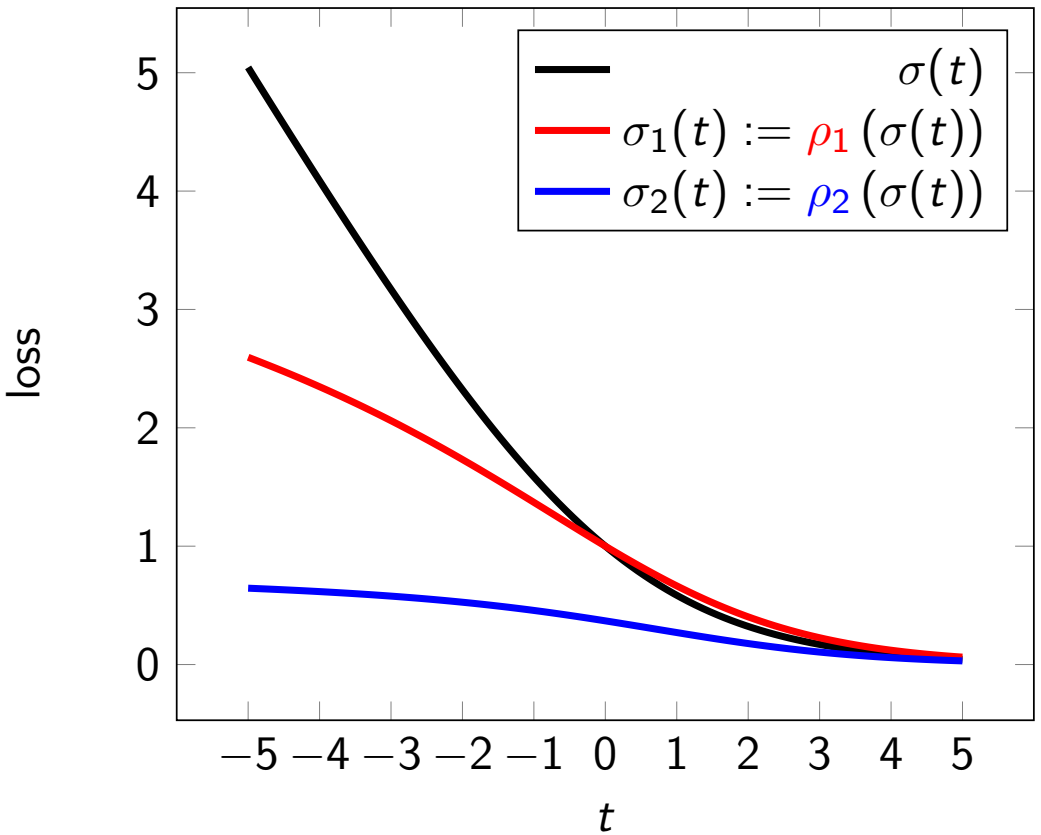
Binary Classification



Convex objective functions are sensitive to outliers

Robust Binary Classification



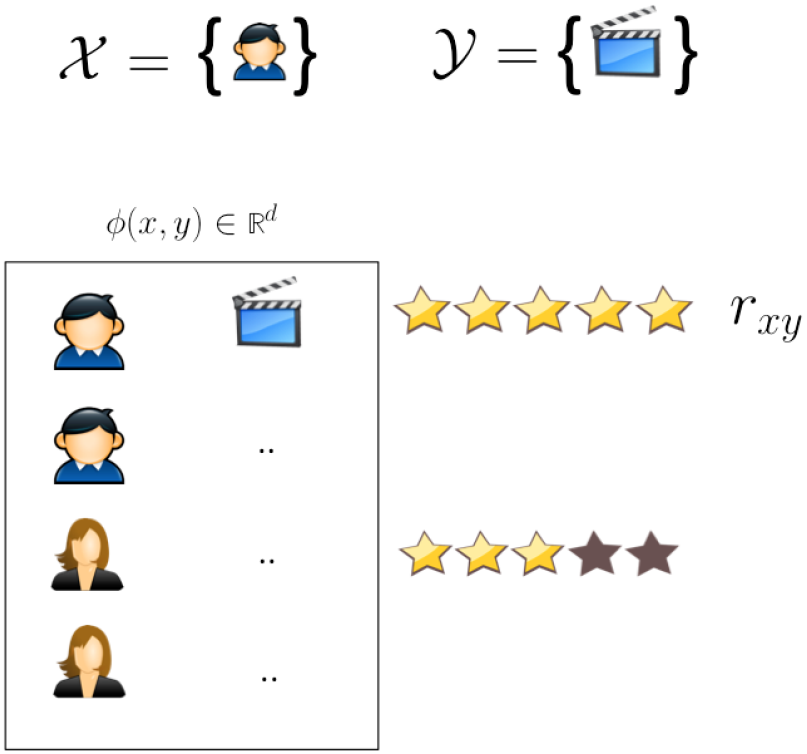
- $\rho_1(t) = \log_2(t+1)$
- $\rho_2(t) = 1 - \frac{1}{\log_2(t+2)}$

Bending the losses provides robustness

Objectives

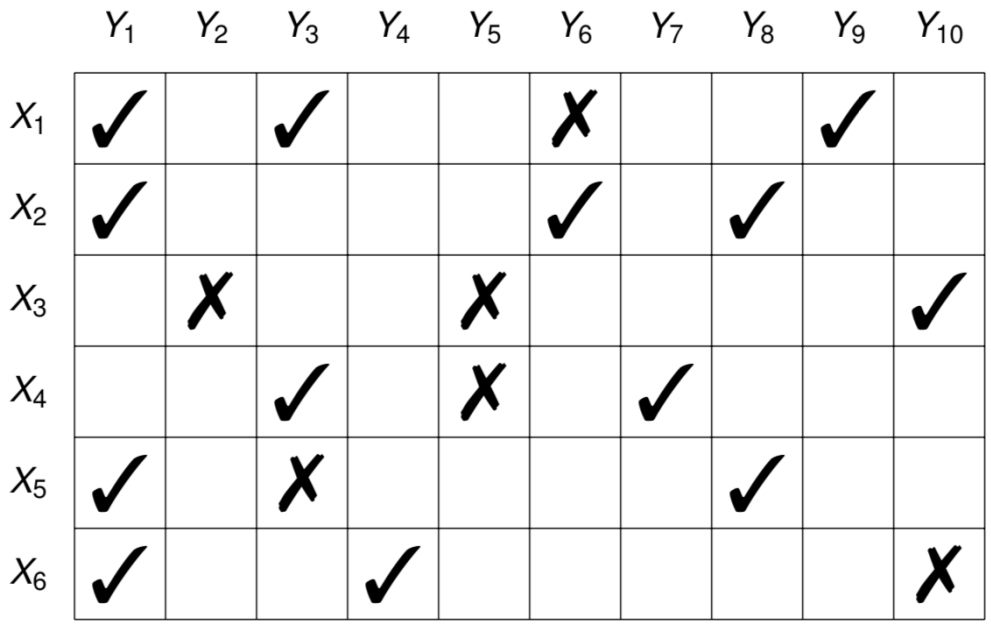
- Can we use this to design robust losses for ranking?
- Two scenarios:
  - **Explicit**: features, labels are observed (**Learning to Rank**)
  - **Implicit**: features, labels are latent (**Latent Collaborative Retrieval**)
- Distributed Optimization Algorithms

Explicit Setting - Learning to Rank



- Scoring function: $f(x, y) = \langle \phi(x, y), \mathbf{w} \rangle$

Implicit Setting - Latent Collaborative Retrieval



- Scoring function: $f(x, y) = \langle U_x, V_y \rangle$

Formulating a Robust Ranking Loss (RoBiRank)

Rank

$$\text{rank}_{\mathbf{w}}(x, y) = \sum_{y' \in \mathcal{Y}_x, y' \neq y} \mathbb{1}\left(f(x, y) - f(x, y') < 0\right)$$

Formulating a Robust Ranking Loss (RoBiRank)

Rank

$$\text{rank}_{\mathbf{w}}(x, y) = \sum_{y' \in \mathcal{Y}_x, y' \neq y} \mathbb{1}\left(f(x, y) - f(x, y') < 0\right)$$

Objective Function

$$L(\mathbf{w}) = \sum_{x \in \mathcal{X}} \sum_{y \in \mathcal{Y}_x} r_{xy} \sum_{y' \in \mathcal{Y}_x, y' \neq y} \sigma\left(f(x, y) - f(x, y')\right)$$

where $\sigma =$ logistic loss.

Formulating a Robust Ranking Loss (RoBiRank)

Applying the robust transformation $\rho_2(t)$

Objective Function

$$L(\mathbf{w}) = \sum_{x \in \mathcal{X}} \sum_{y \in \mathcal{Y}_x} r_{xy} \cdot \rho_2 \left(\sum_{y' \in \mathcal{Y}_x, y' \neq y} \sigma \left(f(x, y) - f(x, y') \right) \right)$$

where $\rho_2(t) = 1 - \frac{1}{\log_2(t+2)}$

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where $\rho_2(t) = 1 - \frac{1}{\log_2(t+2)}$

Directly related to DCG (evaluation metric for ranking).

- minimizing $L(\mathbf{w})$ $\Longrightarrow$ maximizing DCG

Formulating a Robust Ranking Loss (RoBiRank)

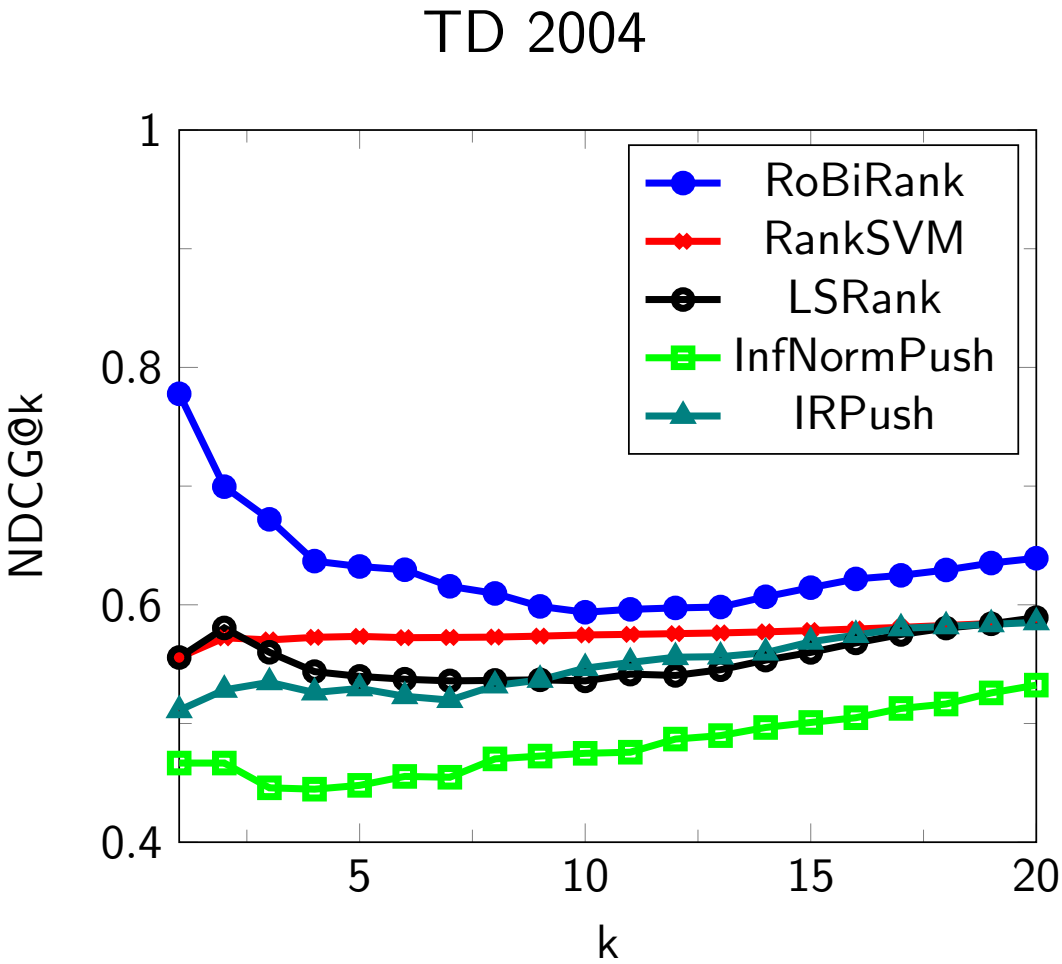
In practice, ρ_1 is easier to optimize,

Objective Function

$$L(\mathbf{w}) = \sum_{x \in \mathcal{X}} \sum_{y \in \mathcal{Y}_x} r_{xy} \cdot \rho_1 \left(\sum_{y' \in \mathcal{Y}_x, y' \neq y} \sigma \left(f(x, y) - f(x, y') \right) \right)$$

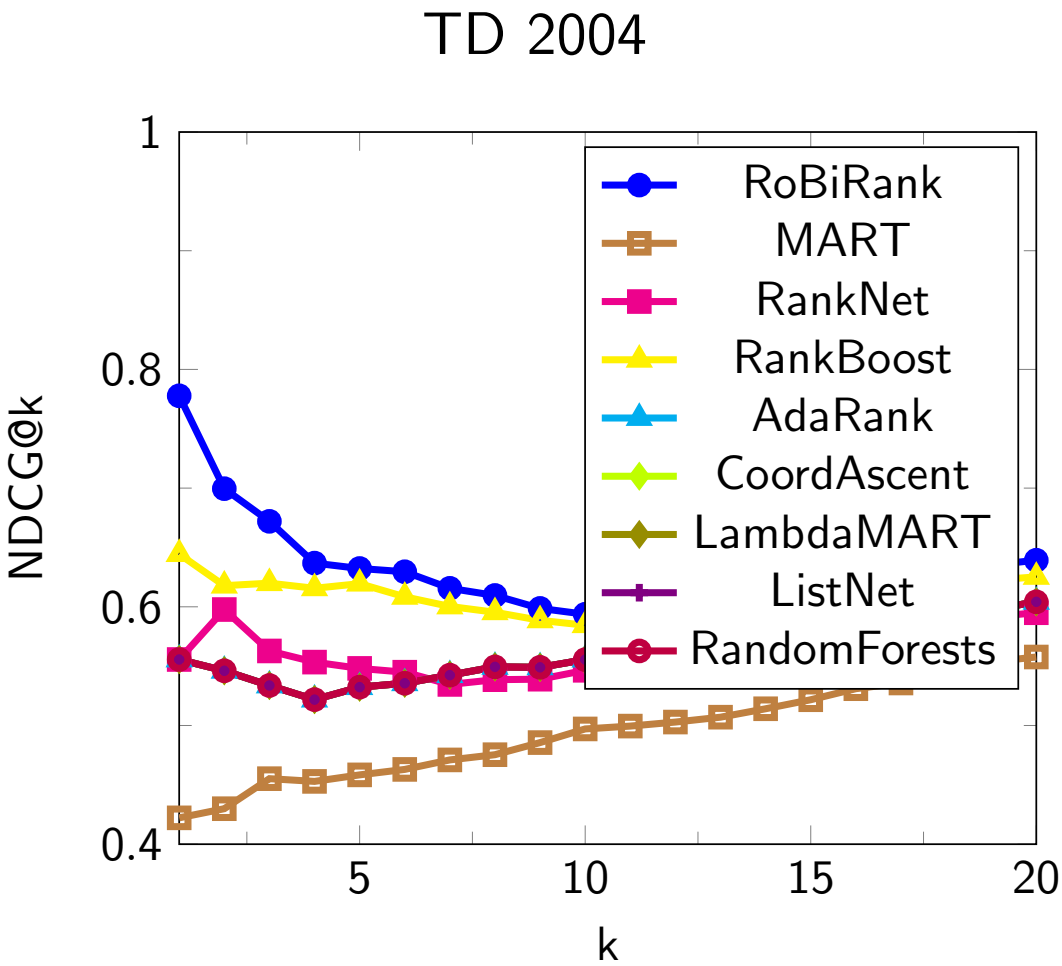
where, $\rho_1(t) = \log_2(t + 1)$

Experiments: Single Machine



RoBiRank shows good accuracy (NDCG) at the top of the ranking list

Experiments: Single Machine



RoBiRank shows good accuracy (NDCG) at the top of the ranking list

How to make RoBiRank Hybrid-Parallel?

Consider RoBiRank with the implicit case (no features, no explicit labels).

Implicit Case

$$\sum_{x \in \mathcal{X}} \sum_{y \in \mathcal{Y}_x} r_{xy} \cdot \rho_1 \underbrace{\left(\sum_{y' \in \mathcal{Y}_x, y' \neq y} \sigma\left(\langle U_x, V_y \rangle - \langle U_x, V_{y'} \rangle \right) \right)}_{\text{expensive}}$$

How can we avoid this?

Linearization

Upper Bound for ρ_1 [Bouchard07]

$$\rho_1(t) = \log_2(t+1) \leq \log_2 \xi + \frac{\xi \cdot (t+1) - 1}{\log_2}$$

The bound holds for any $\xi > 0$ and is tight when $\xi = \frac{1}{t+1}$

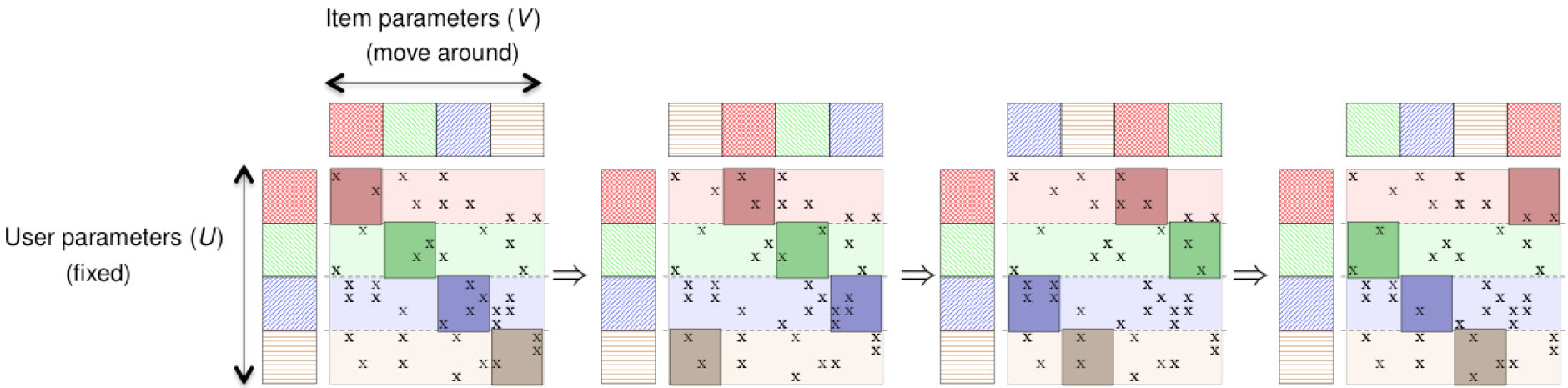
Linearization

We can linearize the objective function using this upper bound,

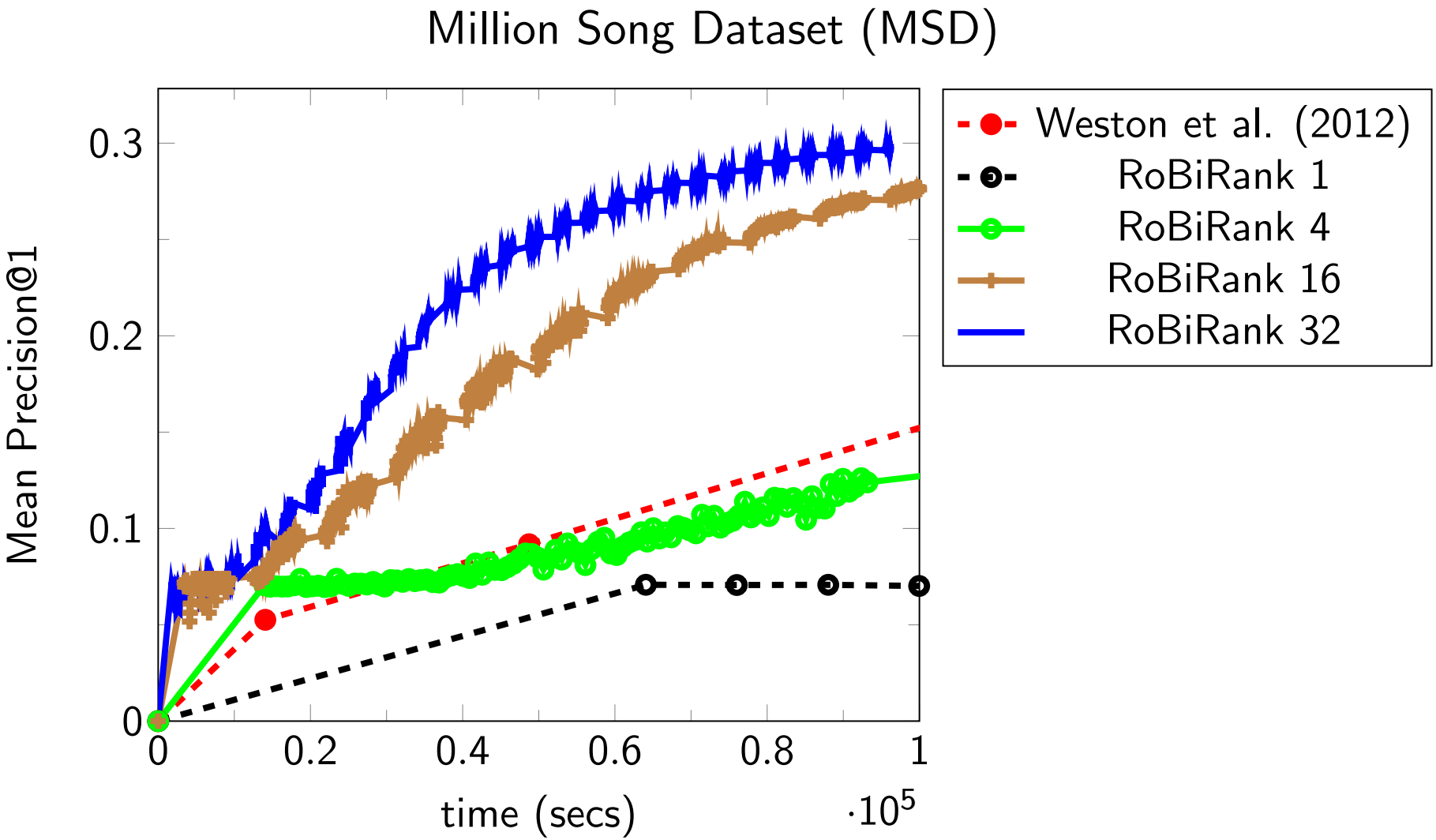
$$\sum_{x \in \mathcal{X}} \sum_{y \in \mathcal{Y}_x} r_{xy} \cdot \left[-\log_2 \xi_{xy} + \frac{\xi_{xy} \cdot \left(\sum_{y' \neq y} \sigma\left(\langle U_x, V_y \rangle - \langle U_x, V_{y'} \rangle \right) + 1 \right) - 1}{\log 2} \right]$$

- Can obtain stochastic gradient updates for U_x, V_y and ξ_{xy}
- ξ_{xy} capture the importance of xy-pair in optimization
- ξ_{xy} also has a closed form solution

Distributing Computation



Experiments: Multi Machine



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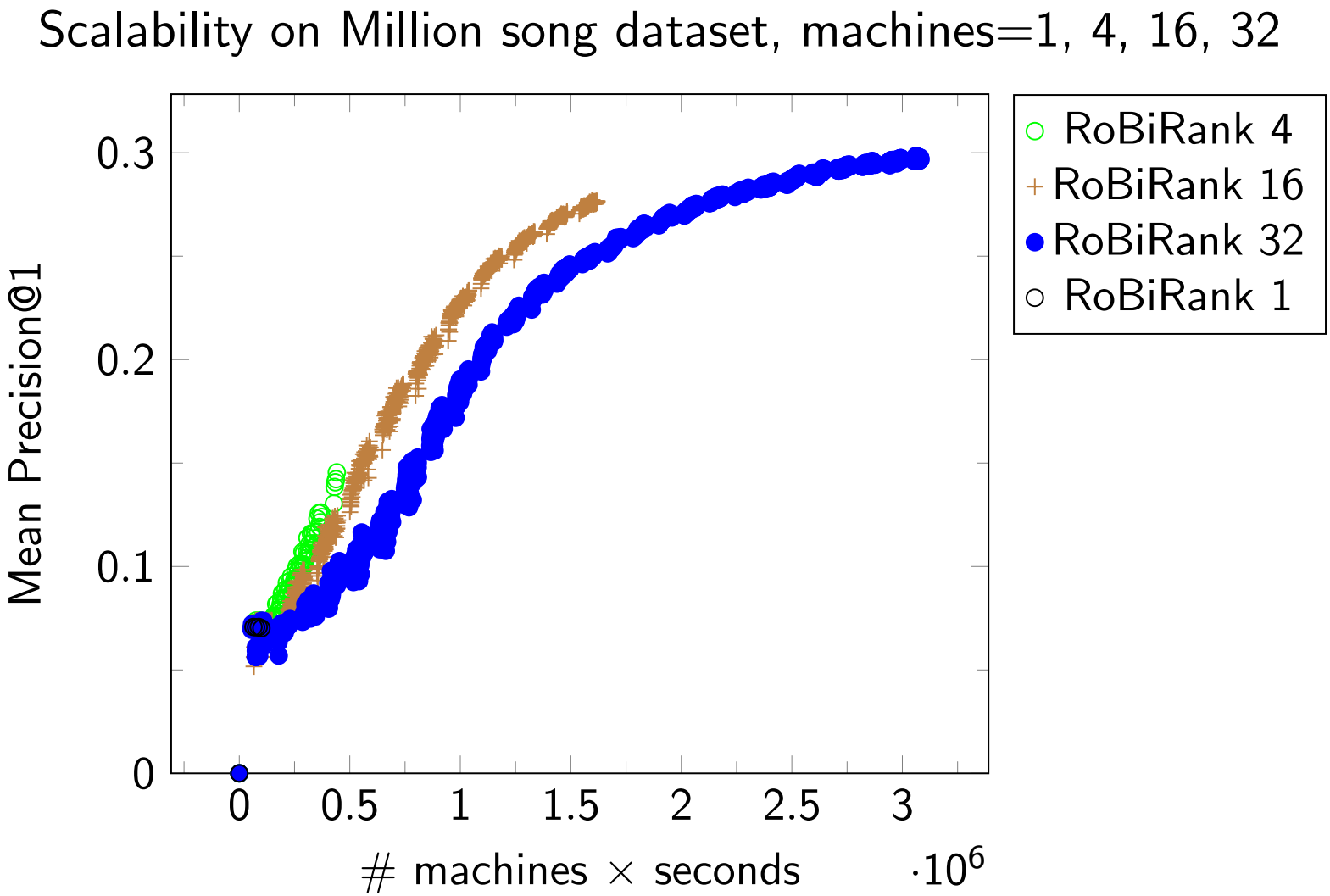

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Conclusion and Key Takeaways

- Data and Model grow hand in hand.

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  - **Distributed, Asynchronous** Algorithms
- Case-studies:
  - Multinomial Logistic Regression
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Future Work: Other Hybrid-Parallel models

- Infinite Mixture Models
  - DP Mixture Model [BleiJordan2006]
  - Pittman-Yor Mixture Model [Dubey et al., 2014]

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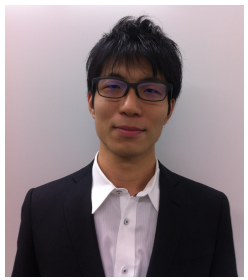
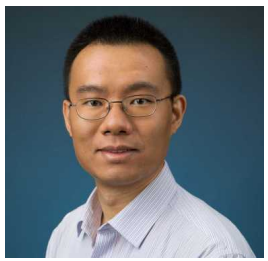
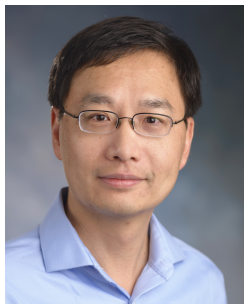
- Deep Neural Networks [Wang et al., 2014]

Acknowledgements

Thanks to all my collaborators.



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- Hyokun Yun, **Parameswaran Raman**, S.V.N. Vishwanathan. *Ranking via Robust Binary Classification*. NIPS 2014.
- Source Code:
  - DS-MLR: `https://bitbucket.org/params/dsmlr`
  - ESVI: `https://bitbucket.org/params/dmixmodels`
  - RoBiRank: `https://bitbucket.org/d_ijk_stra/robirank`

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Polynomial Regression

$$f(\mathbf{x}_i) = w_0 + \sum_{j=1}^{D} w_j \, x_{ij} + \sum_{j=1}^{D} \sum_{j'=j+1}^{D} w_{jj'} \, x_{ij} \, x_{ij'}$$

- $\mathbf{x}_i \in \mathbb{R}^D$ is an example from the dataset $\mathbf{X} \in \mathbb{R}^{N \times D}$
- $w_0 \in \mathbb{R}, \quad \mathbf{w} \in \mathbb{R}^D, \quad \mathbf{W} \in \mathbb{R}^{D \times D}$ are parameters of the model

Polynomial Regression

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- Dense parameterization is not suited for sparse data
- Computationally expensive - $\mathcal{O}\left(N \times D^2\right)$

Factorization Machines

$$f\left(\mathbf{x}_i\right) = w_0 + \sum_{j=1}^{D} w_j \; x_{ij} + \sum_{j=1}^{D} \sum_{j'=j+1}^{D} \left\langle \mathbf{v}_j, \mathbf{v}_{j'} \right\rangle \; x_{ij} \; x_{ij'}$$

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- Factorized Parameterization using $\mathbf{V}\mathbf{V}^T$ instead of Dense $\mathbf{W}$

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- $\mathbf{v}_j \in \mathbb{R}^K$ denotes latent embedding for the j-th feature
- Computationally much cheaper - $\mathcal{O}(N \times D \times K)$

Factorization Machines

$$
\begin{aligned}
f\left(\mathbf{x}_i\right) &= w_0 + \sum_{j=1}^{D} w_j \; x_{ij} + \sum_{j=1}^{D} \sum_{j'=j+1}^{D} \left\langle \mathbf{v}_j, \mathbf{v}_{j'} \right\rangle \; x_{ij} \; x_{ij'} \\
&= w_0 + \sum_{j=1}^{D} w_j \; x_{ij} + \frac{1}{2} \sum_{k=1}^{K} \left\{ \left(\sum_{d=1}^{D} v_{dk} x_{id} \right)^2 - \sum_{j=1}^{D} v_{jk}^2 x_{ij}^2 \right\}
\end{aligned}
$$

Factorization Machines

$$\mathcal{L}(\mathbf{w}, \mathbf{V}) = \frac{1}{N} \sum_{i=1}^{N} l\left(f\left(\mathbf{x}_i\right), y_i\right) + \frac{\lambda_w}{2} \left(\|\mathbf{w}\|_2^2\right) + \frac{\lambda_v}{2} \left(\|\mathbf{V}\|_2^2\right)$$

- λ_w and λ_v are regularizers for $\mathbf{w}$ and $\mathbf{V}$
- $l(\cdot)$ is loss function depending on the task

Factorization Machines

Gradient Descent updates (First-Order Features)

$$
\begin{aligned}
w_j^{t+1} &\leftarrow w_j^t - \eta \sum_{i=1}^{N} \nabla_{w_j} l_i\left(\mathbf{w}, \mathbf{V}\right) + \lambda_w \; w_j^t \\
&= w_j^t - \eta \sum_{i=1}^{N} G_i^t \cdot \nabla_{w_j} f\left(\mathbf{x}_i\right) + \lambda_w \; w_j^t \\
&= w_j^t - \eta \sum_{i=1}^{N} G_i^t \cdot x_{ij} + \lambda_w \; w_j^t
\end{aligned} \tag{1}
$$

where, multiplier G_i^t is given by,

$$
G_i^t = \begin{cases} f\left(x_i\right) - y_i, & \text{if squared loss (regression)} \\ \frac{-y_i}{1+\exp(y_i \cdot f_i(x_i))}, & \text{if logistic loss (classification)} \end{cases} \tag{2}
$$

Factorization Machines

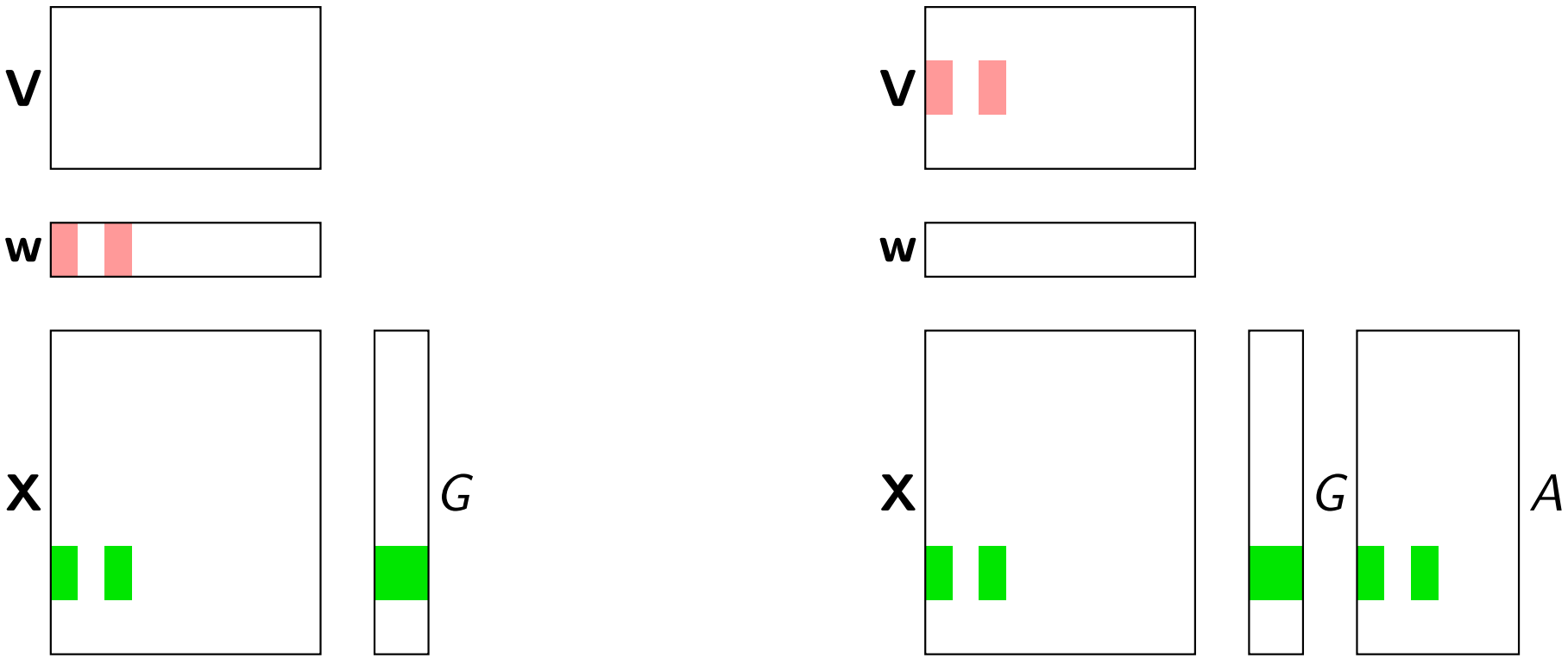
Gradient Descent updates (Second-Order Features)

$$
\begin{aligned}
v_{jk}^{t+1} &\leftarrow v_{jk}^{t} - \eta \sum_{i=1}^{N} \nabla_{v_{jk}} l_i \left(\mathbf{w}, \mathbf{V}\right) + \lambda_v \; v_{jk}^{t} \\
&= v_{jk}^{t} - \eta \sum_{i=1}^{N} G_i^t \cdot \nabla_{v_{jk}} f\left(\mathbf{x}_i\right) + \lambda_v \; v_{jk}^{t} \\
&= v_{jk}^{t} - \eta \sum_{i=1}^{N} G_i^t \cdot \left\{ x_{ij} \left(\sum_{d=1}^{D} v_{dk}^{t} \cdot x_{id} \right) - v_{jk}^{t} \; x_{ij}^2 \right\} + \lambda_v \; v_{jk}^{t} \qquad (3)
\end{aligned}
$$

where, multiplier G_i^t is same as before, synchronization term $a_{ik} = \sum_{d=1}^{D} v_{dk}^{t} \; x_{id}$.

Factorization Machines

Access pattern of parameter updates for w_j and v_{jk}



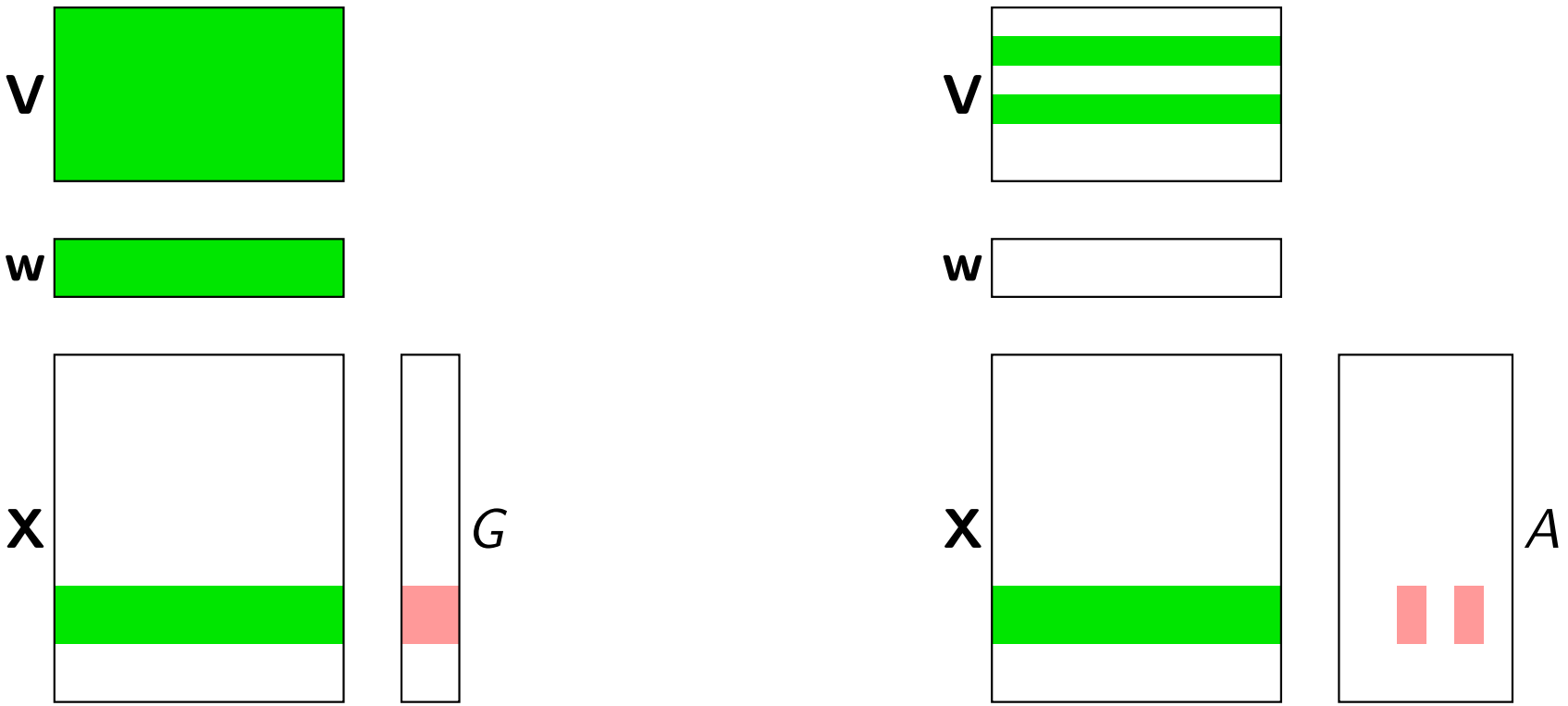
(a) w_j update

(b) v_{jk} update

Figure: Updating w_j requires computing G_i and likewise updating v_{jk} requires computing a_{ik}.

Factorization Machines

Access pattern of parameter updates for w_j and v_{jk}



(a) computing G_i (b) computing a_{ik}

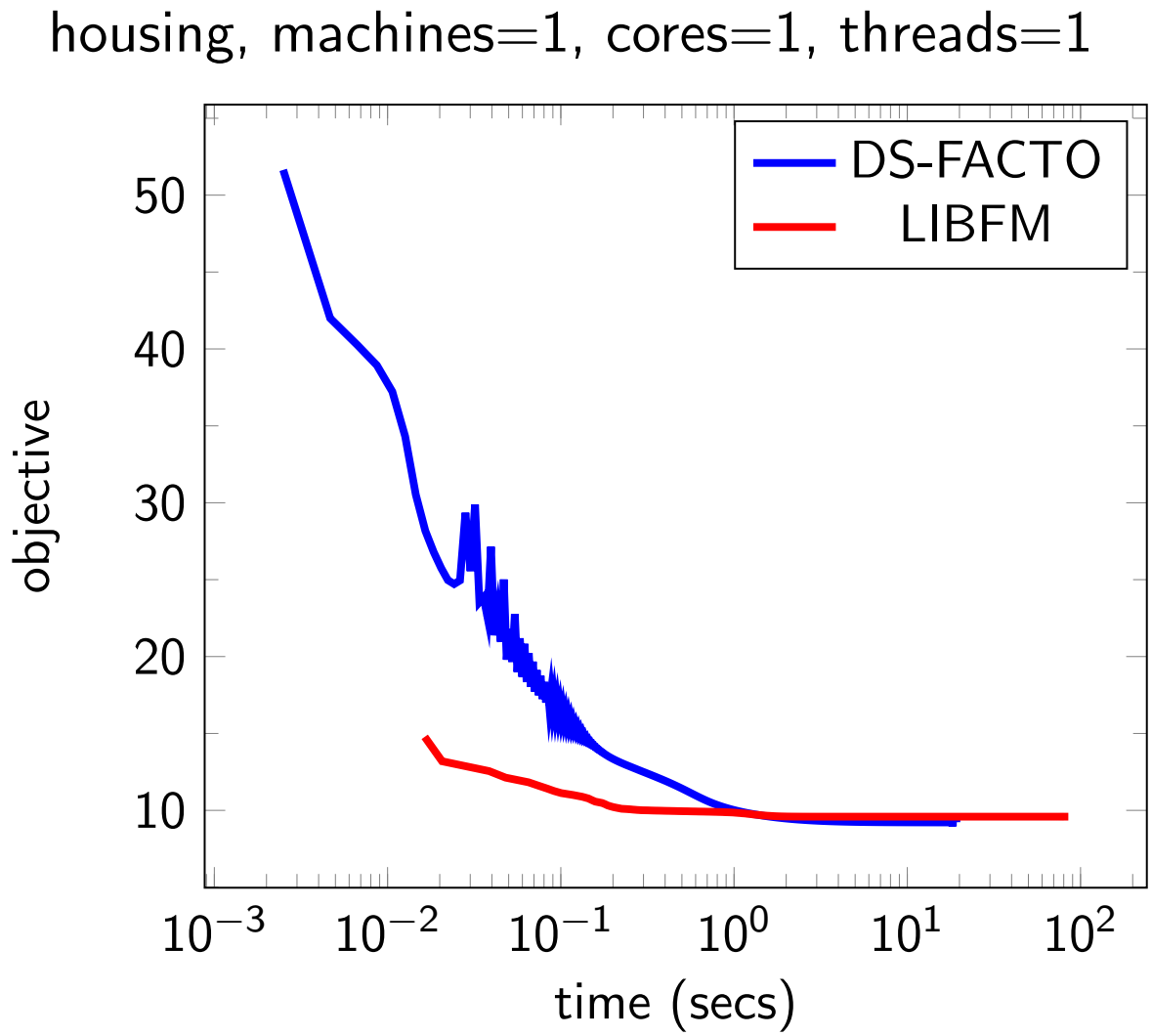
Figure: Computing both G and A requires accessing all the dimensions $j = 1, \ldots, D$. This is the main synchronization bottleneck.

Doubly-Separable Factorization Machines (DS-FACTO)

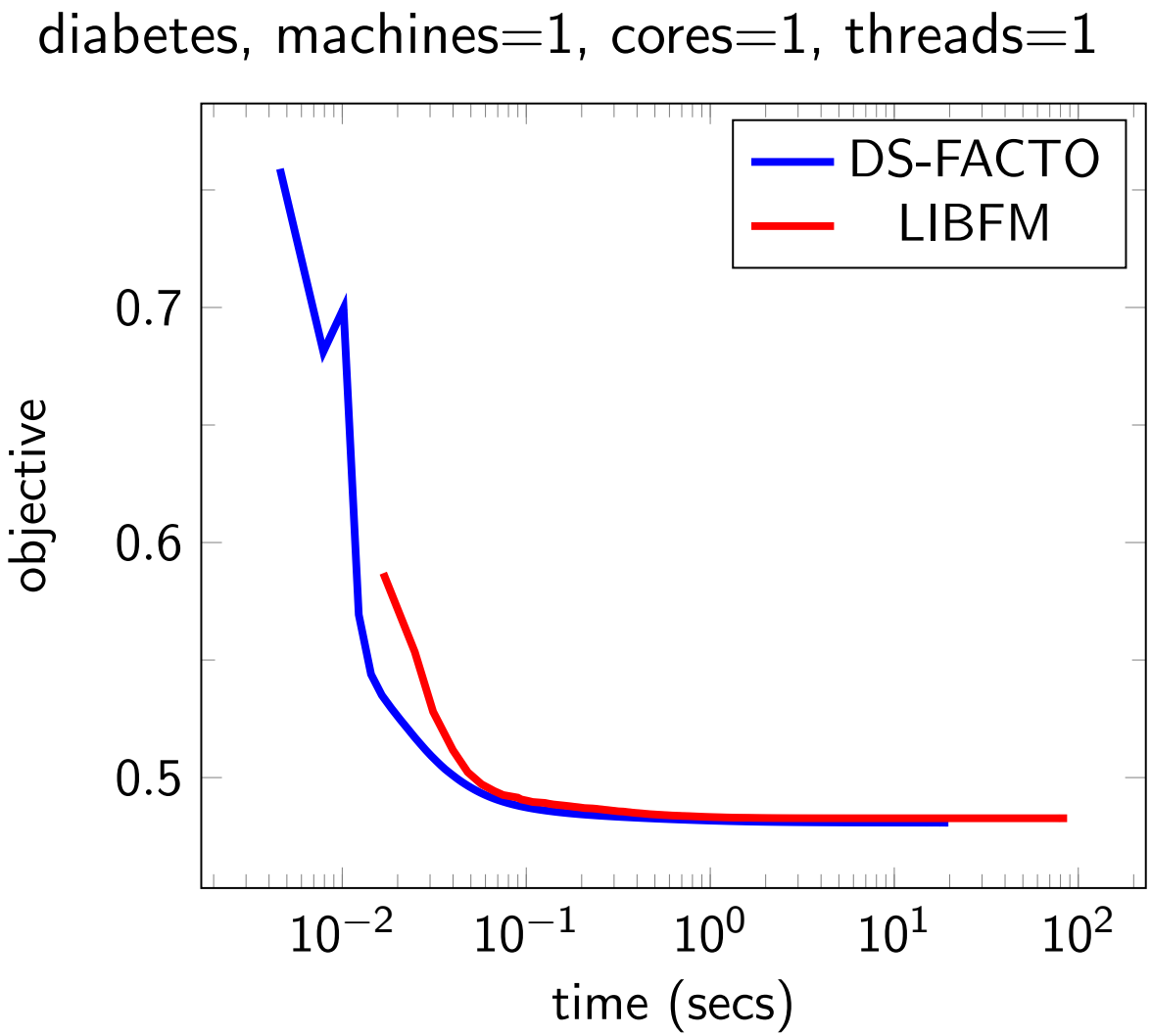
Avoiding bulk-synchronization

- Perform a round of Incremental synchronization after all D updates

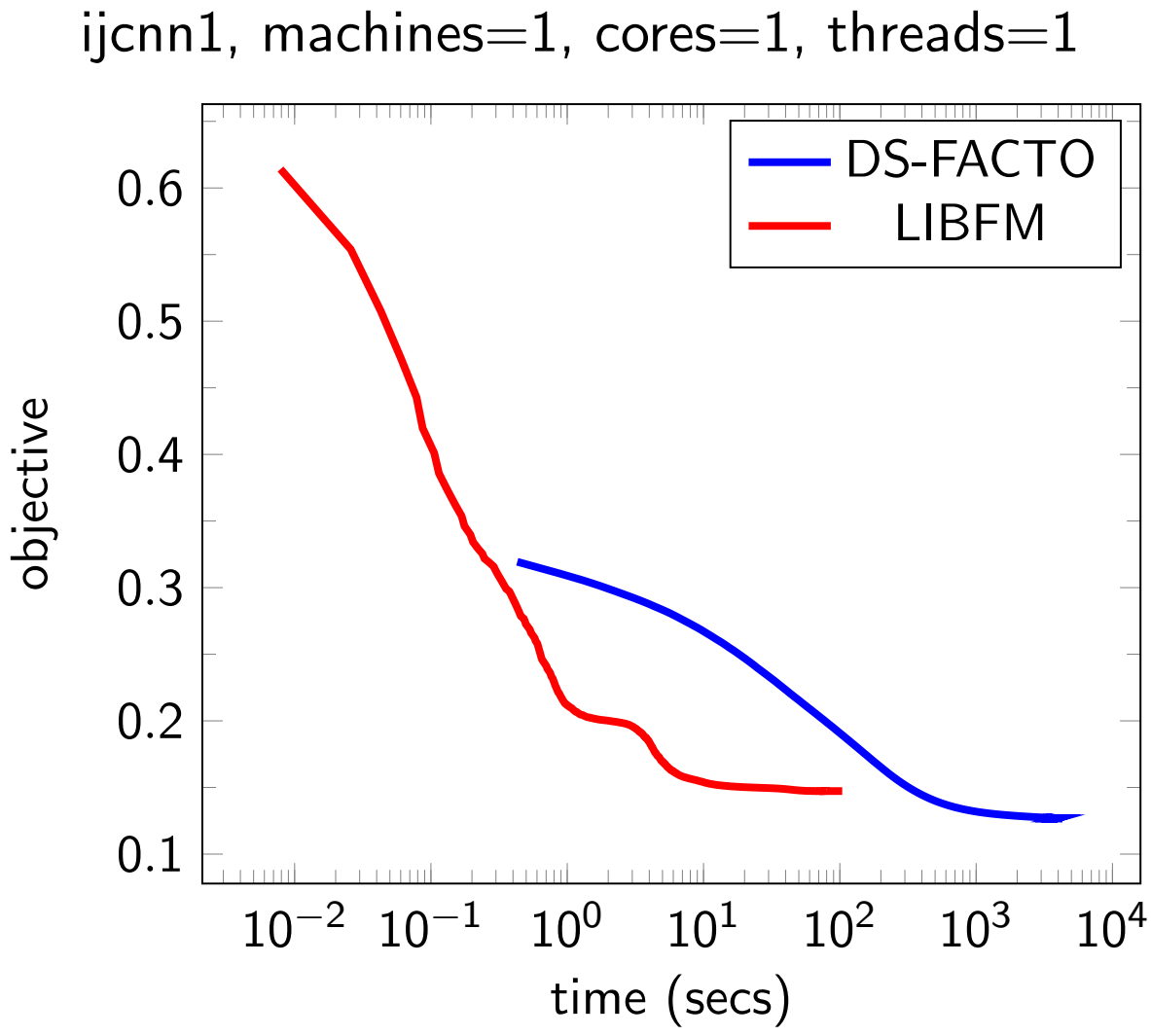
Experiments: Single Machine



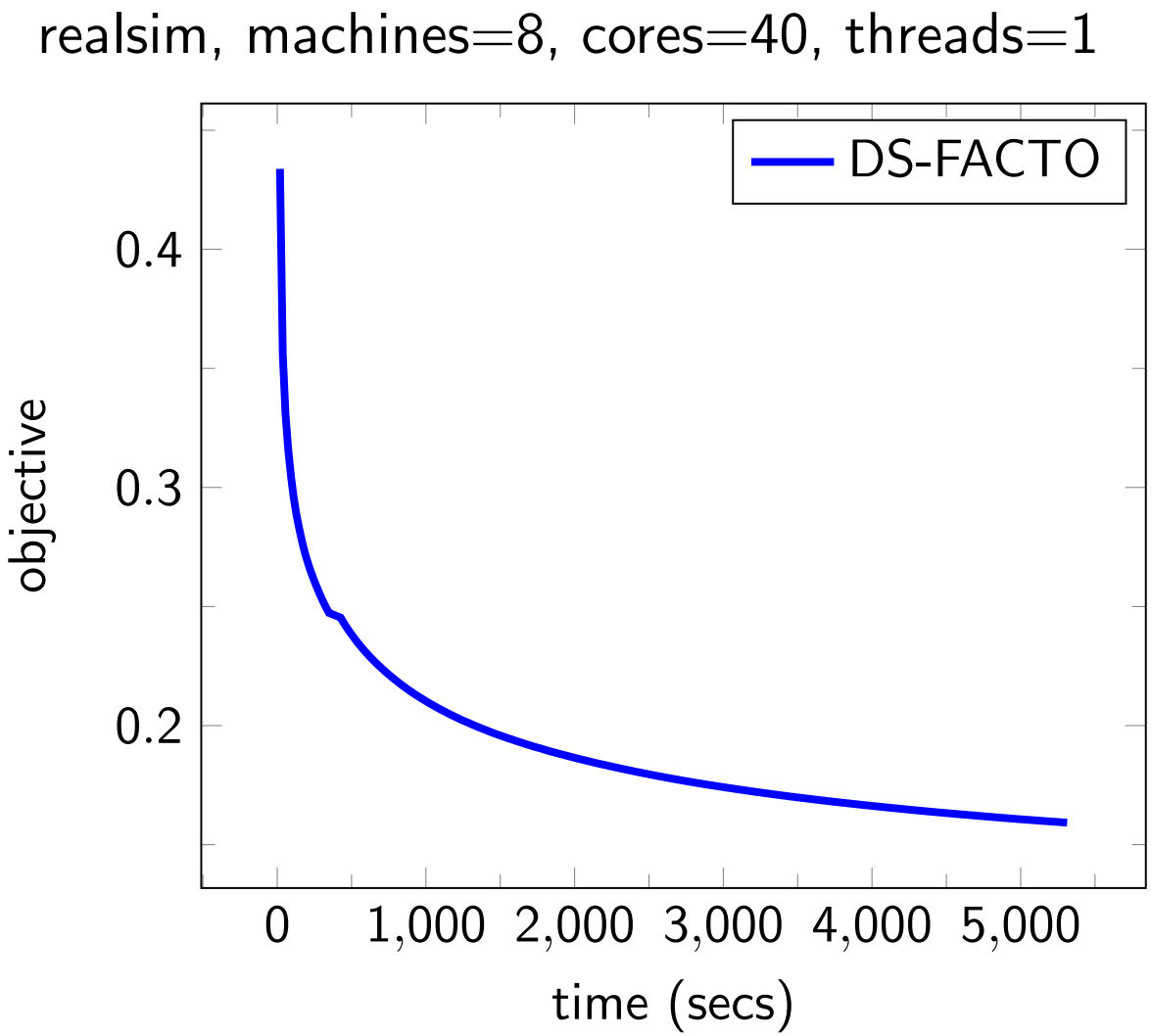
Experiments: Single Machine



Experiments: Single Machine

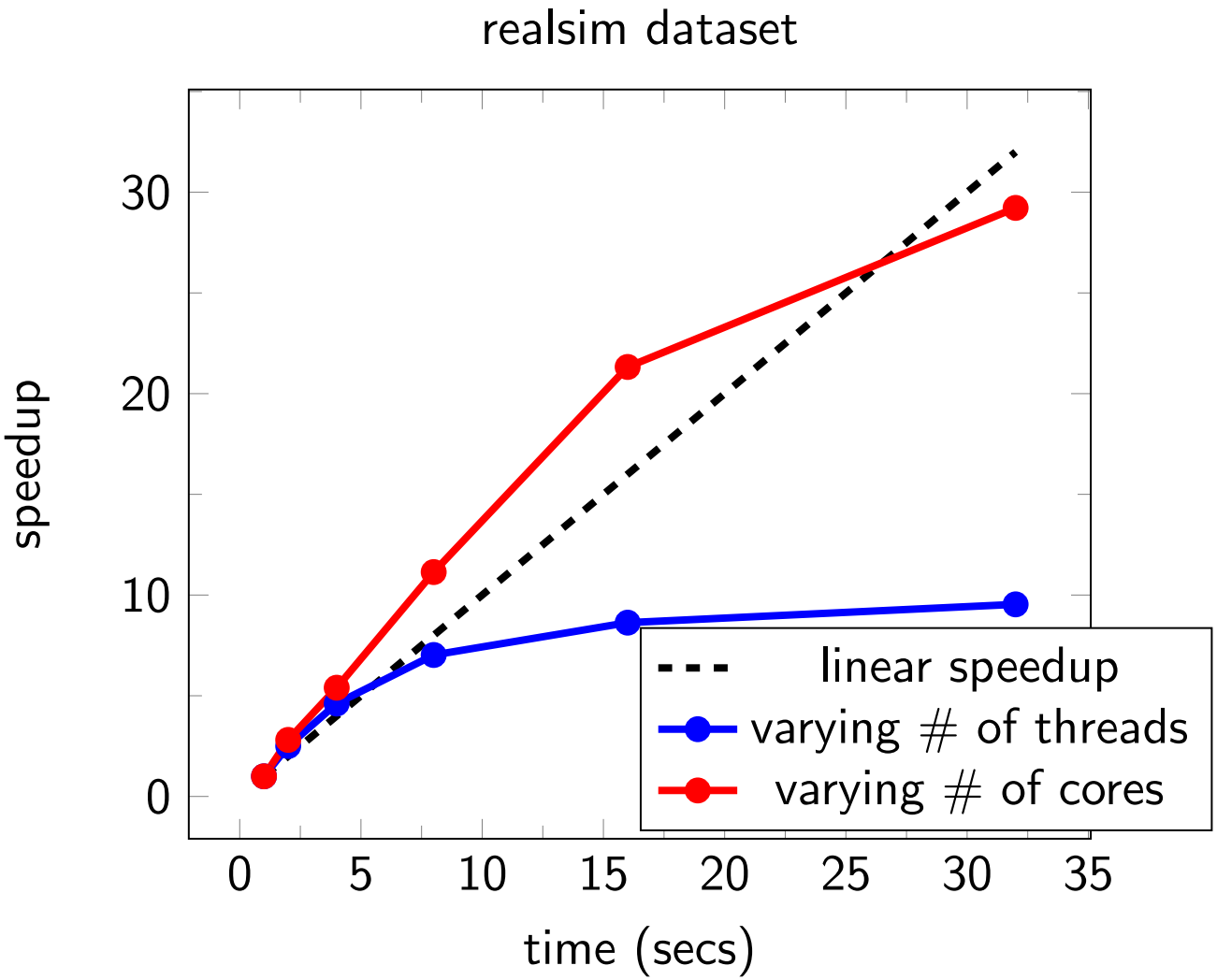


Experiments: Multi Machine



Experiments: Scaling in terms of threads

realsim, Varying cores and threads as 1, 2, 4, 8, 16, 32



Appendix: Alternative Reformulation for DS-MLR

Step 1: Introduce redundant constraints (new parameters A) into the original MLR problem

$$\min_{W,A} \quad L_1(W, A) = \frac{\lambda}{2} \sum_{k=1}^{K} \|\mathbf{w}_k\|^2 - \frac{1}{N} \sum_{i=1}^{N} \sum_{k=1}^{K} y_{ik} \mathbf{w}_k{}^T \mathbf{x}_i - \frac{1}{N} \sum_{i=1}^{N} \log a_i$$

$$\text{s.t.} \quad a_i = \frac{1}{\sum_{k=1}^{K} \exp(\mathbf{w}_k{}^T \mathbf{x}_i)}$$

Appendix: Alternative Reformulation for DS-MLR

Step 2: Turn the problem to unconstrained min-max problem by introducing Lagrange multipliers $\beta_i, \forall i = 1, \ldots, N$

$$\min_{W,A} \max_{\beta} \quad L_2(W, A, \beta) = \frac{\lambda}{2} \sum_{k=1}^{K} \|\mathbf{w}_k\|^2 - \frac{1}{N} \sum_{i=1}^{N} \sum_{k=1}^{K} y_{ik} \mathbf{w}_k{}^T \mathbf{x}_i - \frac{1}{N} \sum_{i=1}^{N} \log a_i + \frac{1}{N} \sum_{i=1}^{N} \sum_{k=1}^{K} \beta_i \, a_i \exp(\mathbf{w}_k{}^T \mathbf{x}_i) - \frac{1}{N} \sum_{i=1}^{N} \beta_i$$

Appendix: Alternative Reformulation for DS-MLR

Step 2: Turn the problem to unconstrained min-max problem by introducing Lagrange multipliers $\beta_i, \forall i = 1, \ldots, N$

$$\min_{W,A} \max_{\beta} \quad L_2(W, A, \beta) = \frac{\lambda}{2} \sum_{k=1}^{K} \|\mathbf{w}_k\|^2 - \frac{1}{N} \sum_{i=1}^{N} \sum_{k=1}^{K} y_{ik} \mathbf{w}_k{}^T \mathbf{x}_i - \frac{1}{N} \sum_{i=1}^{N} \log a_i + \frac{1}{N} \sum_{i=1}^{N} \sum_{k=1}^{K} \beta_i \, a_i \exp(\mathbf{w}_k{}^T \mathbf{x}_i) - \frac{1}{N} \sum_{i=1}^{N} \beta_i$$

Primal Updates for W, A and Dual Update for β (similar in spirit to dual-decomp. methods).

Appendix: Alternative Reformulation for DS-MLR

Step 3: Stare at the updates long-enough!

- When a_i^{t+1} is solved to optimality, it admits an exact closed-form solution given by $a_i^* = \frac{1}{\beta_i \sum_{k=1}^{K} \exp(\mathbf{w}_k{}^T \mathbf{x}_i)}$.
- Dual-ascent update for β_i is no longer needed, since the penalty is always zero if β_i is set to a constant equal to 1.

$$\min_{W,A} \quad L_3(W, A) = \frac{\lambda}{2} \sum_{k=1}^{K} \|\mathbf{w}_k\|^2 - \frac{1}{N} \sum_{i=1}^{N} \sum_{k=1}^{K} y_{ik} \mathbf{w}_k{}^T \mathbf{x}_i - \frac{1}{N} \sum_{i=1}^{N} \log a_i + \frac{1}{N} \sum_{i=1}^{N} \sum_{k=1}^{K} a_i \exp(\mathbf{w}_k{}^T \mathbf{x}_i) - \frac{1}{N}$$

Appendix: Alternative Reformulation for DS-MLR

Step 4: Simple re-write

Doubly-Separable form

$$\min_{W,A} \sum_{i=1}^{N}\sum_{k=1}^{K}\left(\frac{\lambda\|\mathbf{w}_k\|^2}{2N} - \frac{y_{ik}\mathbf{w}_k{}^T\mathbf{x}_i}{N} - \frac{\log a_i}{NK} + \frac{\exp(\mathbf{w}_k{}^T\mathbf{x}_i + \log a_i)}{N} - \frac{1}{NK}\right)$$